科目名	微積分学	対象	1OB-AB	学部 研究科	理学部第一部	学科 専攻科		学籍 番号	
平成 27 年 1 月 26 日 (月) 2 回目 (～ 時限目)				担当	石川 学	学年		氏名	
試験 時間	60 分	注意 事項	① 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )						

評 点

平成 26 年度後期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ $f(x,y) = x^3 + y^3 + \dfrac{7}{4}x^2 + \dfrac{7}{4}y^2 + \dfrac{3}{2}xy$ について，次の問いに答えよ．

(1) $f(x,y)$ の停留点を求めよ．

(2) $f(x,y)$ の極値を求めよ．

(3) $x^2y + xy^2 - xy = 1$ に制限した $f(x,y)$ が点 $(1,1)$ で極値をとるかどうか調べよ．

$\boxed{2}$ 次の積分を求めよ．

(1) $\displaystyle\int_{-1}^{2} \left\{ \int_{2x-1}^{x^2} (2y - 3x)\,dy \right\} dx$

(2) $\displaystyle\int_{2}^{8} \left(\int_{2}^{x} \frac{1}{x^2 y}\,dy \right) dx$

(3) $\displaystyle\int_{0}^{\frac{15}{7}} \left(\int_{\frac{7}{15}x}^{1} y e^{-y^3}\,dy \right) dx$ （順序変更）

(4) $\displaystyle\int\!\!\int_D \frac{y^3}{x^3}\,dxdy \quad \left(D : 1 \leqq x^2 + y^2 \leqq 4,\ -\sqrt{3}x \leqq y \leqq \frac{x}{\sqrt{3}} \right)$

(5) $\displaystyle\int\!\!\int_D x(4x - 5y)\,dxdy \quad (D : x \leqq x^2 + y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0)$

東京理科大学　　年　月　日 **試　験　答　案**　　評　点

科目名		担当	先 生
学籍番号 所属	学部　学科　番	氏名	H26 後・定

1

$$f(x,y)=x^3+y^3+\frac{7}{4}x^2+\frac{7}{4}y^2+\frac{3}{2}xy$$

(1) $\begin{cases} f_x(x,y)=3x^2+\frac{7}{2}x+\frac{3}{2}y=0 & \cdots ① \\ f_y(x,y)=3y^2+\frac{7}{2}y+\frac{3}{2}x=0 & \cdots ② \end{cases}$

① − ② より　$3(x-y)(x+y)+2(x-y)=0$

$(x-y)\{3(x+y)+2\}=0$

$\therefore x=y \quad \vee \quad x+y=-\frac{2}{3}$

(i) $x=y$ のとき，① より

$3x^2+5x=0$

$x(3x+5)=0 \qquad \therefore x=0,-\frac{5}{3}$

(ii) $x+y=-\frac{2}{3}$ のとき，①+② より

$3\{(x+y)^2-2xy\}+5(x+y)=0$

$3\left(\frac{4}{9}-2xy\right)-\frac{10}{3}=0 \qquad \therefore xy=-\frac{1}{3}$

これらより，x,y は t の2次方程式 $t^2+\frac{2}{3}t-\frac{1}{3}=0$ の解である．
実際に解くと

$3t^2+2t-1=0$

$(t+1)(3t-1)=0 \qquad \therefore t=-1,\frac{1}{3}$

以上(i)，(ii)より，停留点は

$(0,0)$，$\left(-\frac{5}{3},-\frac{5}{3}\right)$，$\left(-1,\frac{1}{3}\right)$，$\left(\frac{1}{3},-1\right)$

(2) $f_{xx}(x,y)=6x+\frac{7}{2}$，$f_{yy}(x,y)=6y+\frac{7}{2}$，$f_{xy}(x,y)=\frac{3}{2}$

$H(x,y)=\left(6x+\frac{7}{2}\right)\left(6y+\frac{7}{2}\right)-\left(\frac{3}{2}\right)^2$

・$H(0,0)=\frac{7}{2}\cdot\frac{7}{2}-\left(\frac{3}{2}\right)^2=10>0$，$f_{xx}(0,0)=\frac{7}{2}>0$

$\therefore f(0,0)=0$ ：極小値

・$H\left(-\frac{5}{3},-\frac{5}{3}\right)=\left(-\frac{13}{2}\right)\cdot\left(-\frac{13}{2}\right)-\left(\frac{3}{2}\right)^2=40>0$，$f_{xx}\left(-\frac{5}{3},-\frac{5}{3}\right)=-\frac{13}{2}<0$

$\therefore f\left(-\frac{5}{3},-\frac{5}{3}\right)=\frac{125}{27}$ ：極大値

・$H\left(-1,\frac{1}{3}\right)=\left(-\frac{5}{2}\right)\cdot\frac{11}{2}-\left(\frac{3}{2}\right)^2=-16<0$

$\therefore f\left(-1,\frac{1}{3}\right)$ ：極値でない

・$H\left(\frac{1}{3},-1\right)=\frac{11}{2}\cdot\left(-\frac{5}{2}\right)-\left(\frac{3}{2}\right)^2=-16<0$

$\therefore f\left(\frac{1}{3},-1\right)$ ：極値でない．

(3) $g(x,y)=x^2y+xy^2-xy-1$，$P: g(x,y)=0$ とおく．
$g_y(x,y)=x^2+2xy-x$ だから，$g(1,1)=0$，$g_y(1,1)=2\neq 0$ である．
よって，陰関数定理より，点 $(1,1)$ のある開近傍 U 内では
$g(x,y)=0$ における y は x の C^∞ 級関数 $y=\varphi(x)$ となり

$\varphi(1)=1$，$x^2\varphi(x)+x\varphi(x)^2-x\varphi(x)=1 \quad \cdots ③$

をみたす．

③ の両辺を x で微分して

$2x\cdot\varphi(x)+x^2\cdot\varphi'(x)+1\cdot\varphi(x)^2+x\{2\varphi(x)\cdot\varphi'(x)\}-1\cdot\varphi(x)-x\cdot\varphi'(x)=0$

$2x\varphi(x)+x^2\varphi'(x)+\varphi(x)^2+2x\varphi(x)\varphi'(x)-\varphi(x)-x\varphi'(x)=0 \quad \cdots ④$

$x=1$ を代入して

$2\cdot1\cdot1+1^2\cdot\varphi'(1)+1^2+2\cdot1\cdot1\cdot\varphi'(1)-1-1\cdot\varphi'(1)=0$

$\therefore \varphi'(1)=-1$

④の両辺を x で微分して

$2\cdot\varphi(x)+2x\cdot\varphi'(x)+2x\cdot\varphi'(x)+x^2\cdot\varphi''(x)+2\varphi(x)\cdot\varphi'(x)$
$+2\cdot\varphi(x)\varphi'(x)+2x\cdot\varphi'(x)\cdot\varphi'(x)+2x\varphi(x)\cdot\varphi''(x)-\varphi'(x)$
$-1\cdot\varphi'(x)-x\varphi''(x)=0$

$2\varphi(x)+4x\varphi'(x)+x^2\varphi''(x)+4\varphi(x)\varphi'(x)+2x\varphi'(x)^2$
$+2x\varphi(x)\varphi''(x)-2\varphi'(x)-x\varphi''(x)=0$

$x=1$ を代入して

$2\cdot1+4\cdot1\cdot(-1)+1^2\cdot\varphi''(1)+4\cdot1\cdot(-1)+2\cdot1\cdot(-1)^2$
$+2\cdot1\cdot1\cdot\varphi''(1)-2\cdot(-1)-1\cdot\varphi''(1)=0$

$\therefore \varphi''(1)=1$

さて，U 内では

$f|_P(x,y)=f(x,\varphi(x))$

$=x^3+\varphi(x)^3+\frac{7}{4}x^2+\frac{7}{4}\varphi(x)^2+\frac{3}{2}x\varphi(x)$

となるから，これを $F(x)$ とおき，$F(x)$ が $x=1$ で極値をとるかどうか調べればよい．

$F(x)=x^3+\varphi(x)^3+\frac{7}{4}x^2+\frac{7}{4}\varphi(x)^2+\frac{3}{2}x\varphi(x)$

$F'(x)=3x^2+3\varphi(x)^2\cdot\varphi'(x)+\frac{7}{2}x+\frac{7}{2}\varphi(x)\cdot\varphi'(x)+\frac{3}{2}\cdot\varphi(x)+\frac{3}{2}x\cdot\varphi'(x)$

$=3x^2+3\varphi(x)^2\varphi'(x)+\frac{7}{2}x+\frac{7}{2}\varphi(x)\varphi'(x)+\frac{3}{2}\varphi(x)+\frac{3}{2}x\varphi'(x)$

だから

$F'(1)=3\cdot1^2+3\cdot1^2\cdot(-1)+\frac{7}{2}\cdot1+\frac{7}{2}\cdot1\cdot(-1)+\frac{3}{2}\cdot1+\frac{3}{2}\cdot1\cdot(-1)=0$

また

$F''(x)=6x+\{6\varphi(x)\cdot\varphi'(x)\}\cdot\varphi'(x)+3\varphi(x)^2\cdot\varphi''(x)+\frac{7}{2}$
$+\frac{7}{2}\varphi'(x)\cdot\varphi'(x)+\frac{7}{2}\varphi(x)\cdot\varphi''(x)+\frac{3}{2}\varphi'(x)+\frac{3}{2}\cdot\varphi'(x)+\frac{3}{2}x\cdot\varphi''(x)$

$=6x+6\varphi(x)\varphi'(x)^2+3\varphi(x)^2\varphi''(x)+\frac{7}{2}+\frac{7}{2}\varphi'(x)^2$
$+\frac{7}{2}\varphi(x)\varphi''(x)+3\varphi'(x)+\frac{3}{2}x\varphi''(x)$

だから

$F''(1)=6\cdot1+6\cdot1\cdot(-1)^2+3\cdot1^2\cdot1+\frac{7}{2}+\frac{7}{2}\cdot(-1)^2$
$+\frac{7}{2}\cdot1\cdot1+3\cdot(-1)+\frac{3}{2}\cdot1\cdot1$

$=24>0$

$\therefore F(1)=7$ ：極小値

$\boxed{2}$

(1) $\displaystyle\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(2y-3x)\,dy\right\}dx$

$\displaystyle = \int_{-1}^{2}\left[y^2-3xy\right]_{y=2x-1}^{y=x^2}dx$

$\displaystyle = \int_{-1}^{2}\left[(x^4-3x^3)-\{(2x-1)^2-3x(2x-1)\}\right]dx$

$\displaystyle = \int_{-1}^{2}(x^4-3x^3+2x^2+x-1)\,dx$

$\displaystyle = \left[\frac{1}{5}x^5-\frac{3}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2-x\right]_{-1}^{2}$

$\displaystyle = \frac{1}{5}\{32-(-1)\}-\frac{3}{4}(16-1)+\frac{2}{3}\{8-(-1)\}+\frac{1}{2}(4-1)-\{2-(-1)\}$

$\displaystyle = -\frac{3}{20}$

(2) $\displaystyle\int_{2}^{8}\left(\int_{2}^{x}\frac{1}{x^2y}\,dy\right)dx$

$\displaystyle = \int_{2}^{8}\left[\frac{1}{x^2}\log y\right]_{y=2}^{y=x}dx$

$$\begin{array}{cc} \log x & -\frac{1}{x} \\ \frac{1}{x} & \frac{1}{x^2} \end{array}$$

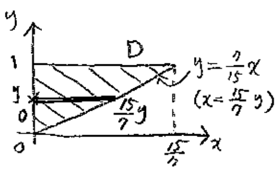
$\displaystyle = \int_{2}^{8}\left(\frac{1}{x^2}\log x-\frac{1}{x^2}\log 2\right)dx$

$\displaystyle = \left[\left(-\frac{1}{x}\log x-\frac{1}{x}\right)+\frac{1}{x}\log 2\right]_{2}^{8}$

$\displaystyle = -\left(\frac{1}{8}\log 8-\frac{1}{2}\log 2\right)-\left(\frac{1}{8}-\frac{1}{2}\right)+\left(\frac{1}{8}-\frac{1}{2}\right)\log 2$

$\displaystyle = \frac{3}{8}-\frac{1}{4}\log 2$

(3) $\displaystyle\int_{0}^{\frac{15}{7}}\left(\int_{\frac{7}{15}x}^{1}y e^{-y^3}\,dy\right)dx \quad \cdots (*)$



積分領域は

$D: 0 \leqq x \leqq \frac{15}{7},\ \frac{7}{15}x \leqq y \leqq 1$

であるが，これは

$D: 0 \leqq y \leqq 1,\ 0 \leqq x \leqq \frac{15}{7}y$

でもあるから

$\displaystyle (*) = \int_{0}^{1}\left(\int_{0}^{\frac{15}{7}y}y e^{-y^3}\,dx\right)dy$

$\displaystyle = \int_{0}^{1}\left[xy e^{-y^3}\right]_{x=0}^{x=\frac{15}{7}y}dy$

$\displaystyle = \int_{0}^{1}\frac{15}{7}y^2e^{-y^3}\,dy$

$\displaystyle = -\frac{5}{7}\int_{0}^{1}e^{-y^3}\cdot(-3y^2)\,dy$

$\displaystyle = -\frac{5}{7}\left[e^{-y^3}\right]_{0}^{1}$

$\displaystyle = -\frac{5}{7}\left(\frac{1}{e}-1\right)$

$\displaystyle = \frac{5}{7}\left(1-\frac{1}{e}\right)$

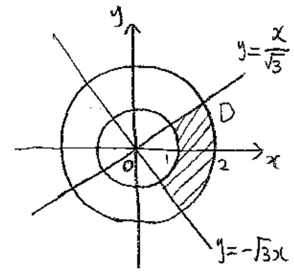
(4), (5) は

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \left(\begin{array}{l} r \geqq 0 \\ \theta : 1\text{周分} \end{array}\right)$

とおく.

(4) $\displaystyle\iint_D \frac{y^3}{x^3}\,dxdy \quad \left(D: 1 \leqq x^2+y^2 \leqq 4,\ -\sqrt{3}x \leqq y \leqq \frac{x}{\sqrt{3}}\right)$

$\displaystyle = \iint_{D'}\frac{r^3\sin^3\theta}{r^3\cos^3\theta}\cdot r\,dr\,d\theta$



$\left(D': 1 \leqq r \leqq 2,\ -\frac{\pi}{3} \leqq \theta \leqq \frac{\pi}{6}\right)$

$\displaystyle = \int_{1}^{2}r\,dr \times \int_{-\frac{\pi}{3}}^{\frac{\pi}{6}}\frac{(1-\cos^2\theta)\sin\theta}{\cos^3\theta}\,d\theta$

$\displaystyle = \left[\frac{1}{2}r^2\right]_{1}^{2} \times \int_{-\frac{\pi}{3}}^{\frac{\pi}{6}}\left\{-(\cos\theta)^{-3}\cdot(-\sin\theta)+\frac{-\sin\theta}{\cos\theta}\right\}d\theta$

$\displaystyle = \frac{1}{2}(4-1) \times \left[-\frac{1}{-2}(\cos\theta)^{-2}+\log|\cos\theta|\right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}}$

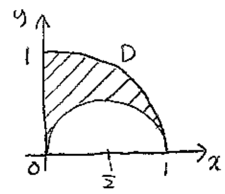
$\displaystyle = \frac{3}{2}\left\{\frac{1}{2}\left(\frac{4}{3}-4\right)+\left(\log\frac{\sqrt{3}}{2}-\log\frac{1}{2}\right)\right\}$

$\displaystyle = \frac{3}{2}\left(-\frac{4}{3}+\log\sqrt{3}\right)$

$\displaystyle = \frac{3}{4}\log 3-2$

(5) $\displaystyle\iint_D x(4x-5y)\,dxdy \quad (D: x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0)$

$\displaystyle = \iint_{D'}r\cos\theta(4r\cos\theta-5r\sin\theta)\cdot r\,dr\,d\theta$



$\left(D': 0 \leqq \theta \leqq \frac{\pi}{2},\ \cos\theta \leqq r \leqq 1\right)$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\left\{\int_{\cos\theta}^{1}r^3(4\cos^2\theta-5\cos\theta\sin\theta)\,dr\right\}d\theta$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\left[\frac{1}{4}r^4(4\cos^2\theta-5\cos\theta\sin\theta)\right]_{r=\cos\theta}^{r=1}d\theta$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\frac{1}{4}(1-\cos^4\theta)(4\cos^2\theta-5\cos\theta\sin\theta)\,d\theta$

$\displaystyle = \frac{1}{4}\int_{0}^{\frac{\pi}{2}}\left\{4\cos^2\theta-4\cos^6\theta+5\cos\theta\cdot(-\sin\theta)-5\cos^5\theta\cdot(-\sin\theta)\right\}d\theta$

$\displaystyle = \frac{1}{4}\left(4\cdot\frac{1}{2}\cdot\frac{\pi}{2}-4\cdot\frac{5\cdot3\cdot1}{6\cdot4\cdot2}\cdot\frac{\pi}{2}+\left[\frac{5}{2}\cos^2\theta-\frac{5}{6}\cos^6\theta\right]_{0}^{\frac{\pi}{2}}\right)$

$\displaystyle = \frac{1}{4}\left\{\frac{3}{8}\pi+\frac{5}{2}(0-1)-\frac{5}{6}(0-1)\right\}$

$\displaystyle = \frac{1}{4}\left(\frac{3}{8}\pi-\frac{5}{3}\right)$

$\displaystyle = \frac{3}{32}\pi-\frac{5}{12}$