科目名	微積分学B 微積分学	対象	1OB-AB	学部 研究科	理学部第一部	学科 専攻科		学籍 番号	
平成 28 年 2 月 1 日 (月) 2 回目 (〜 時限目)				担当	石川 学	学年		氏名	
試験 時間	60 分	注意 事項	① 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )						

評 点

平成 27 年度後期定期試験

※解答用紙の裏面使用可

[1] $x^3y+xy^3-xy=1$ に制限した $f(x,y)=x^3+y^3+\dfrac{3}{2}x^2+\dfrac{3}{2}y^2+2xy$ が点 $(1,1)$ で極値をとるかどうか調べよ．

[2] 次の積分を求めよ．

(1) $\displaystyle\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(2x-y)dy\right\}dx$

(2) $\displaystyle\int_{2}^{3}\left(\int_{x-1}^{x^2}\frac{x}{y}dy\right)dx$

(3) $\displaystyle\int_{0}^{2}\left(\int_{\frac{x}{2}}^{1}y^2e^{-y^4}dy\right)dx$ （順序変更）

(4) $\displaystyle\int\int_D\frac{y}{x^3}dxdy \quad \left(D:1\leqq x^2+y^2\leqq 4,\ -\sqrt{3}\,x\leqq y\leqq\frac{x}{\sqrt{3}}\right)$

(5) $\displaystyle\int\int_D x(8x-7y)dxdy \quad (D:x\leqq x^2+y^2\leqq 1,\ x\geqq 0,\ y\geqq 0)$

1

$f(x,y)=x^3+y^3+\dfrac{3}{2}x^2+\dfrac{3}{2}y^2+2xy$ より

$$f_x(x,y)=3x^2+3x+2y,\ f_y(x,y)=3y^2+3y+2x$$

$$f_{xx}(x,y)=6x+3,\ f_{yy}(x,y)=6y+3,\ f_{xy}(x,y)=2$$

また，$g(x,y)=x^3y+xy^3-xy-1$ とおくと

$$g_x(x,y)=3x^2y+y^3-y,\ g_y(x,y)=x^3+3xy^2-x$$

$$g_{xx}(x,y)=6xy,\ g_{yy}(x,y)=6xy,\ g_{xy}(x,y)=3x^2+3y^2-1$$

$(x,y)=(1,1)$ のとき

$$f_x=8,\ f_y=8,\ g_x=3,\ g_y=3$$

より，Lagrange の未定乗数 λ は $\lambda=\dfrac{8}{3}$

さらに

$$f_{xx}-\lambda g_{xx}=9-\frac{8}{3}\cdot 6=-7,\ f_{yy}-\lambda g_{yy}=9-\frac{8}{3}\cdot 6=-7,\ f_{xy}-\lambda g_{xy}=2-\frac{8}{3}\cdot 5=-\frac{34}{3}$$

であるから

$$D(1,1)=\begin{vmatrix} 0 & 3 & 3 \\ 3 & -7 & -\frac{34}{3} \\ 3 & -\frac{34}{3} & -7 \end{vmatrix}=-78<0$$

$\therefore\quad f(1,1)=7$：極小値

2

(1)
$$\begin{aligned}\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(2x-y)dy\right\}dx &= \int_{-1}^{2}\left[2xy-\frac{1}{2}y^2\right]_{y=2x-1}^{y=x^2}dx \\ &= \int_{-1}^{2}\left[\left(2x^3-\frac{1}{2}x^4\right)-\left\{2x(2x-1)-\frac{1}{2}(2x-1)^2\right\}\right]dx \\ &= \int_{-1}^{2}\left(-\frac{1}{2}x^4+2x^3-2x^2+\frac{1}{2}\right)dx \\ &= \left[-\frac{1}{10}x^5+\frac{1}{2}x^4-\frac{2}{3}x^3+\frac{1}{2}x\right]_{-1}^{2} \\ &= \left(-\frac{16}{5}+8-\frac{16}{3}+1\right)-\left(\frac{1}{10}+\frac{1}{2}+\frac{2}{3}-\frac{1}{2}\right) \\ &= -\frac{3}{10}\end{aligned}$$

(2)
$$\begin{aligned}\int_{2}^{3}\left(\int_{x-1}^{x^2}\frac{x}{y}dy\right)dx &= \int_{2}^{3}\Big[x\log y\Big]_{y=x-1}^{y=x^2}dx \\ &= \int_{2}^{3}\{2x\log x-x\log(x-1)\}dx \\ &= \left[\left(x^2\log x-\frac{1}{2}x^2\right)-\left\{\frac{1}{2}(x^2-1)\log(x-1)-\frac{1}{4}(x+1)^2\right\}\right]_{2}^{3} \\ &= (9\log 3-4\log 2)-\frac{1}{2}(9-4)-\frac{1}{2}(8\log 2-3\cdot 0)+\frac{1}{4}(16-9) \\ &= 9\log 3-8\log 2-\frac{3}{4}\end{aligned}$$

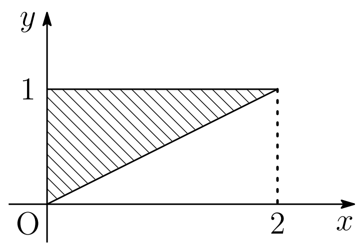
(3) 積分領域は

$D : 0 \leqq x \leqq 2,\ \dfrac{x}{2} \leqq y \leqq 1$

であるが，これは

$D : 0 \leqq y \leqq 1,\ 0 \leqq x \leqq 2y$

でもあるから



$$
\begin{aligned}
\int_0^2 \left(\int_{\frac{x}{2}}^1 y^2 e^{-y^4} dy \right) dx &= \int_0^1 \left(\int_0^{2y} y^2 e^{-y^4} dx \right) dy \\
&= \int_0^1 \left[x y^2 e^{-y^4} \right]_{x=0}^{x=2y} dy \\
&= \int_0^1 2y^3 e^{-y^4} dy \\
&= -\frac{1}{2} \int_0^1 e^{-y^4} \cdot (-4y^3) dy \\
&= -\frac{1}{2} \left[e^{-y^4} \right]_0^1 \\
&= -\frac{1}{2} \left(\frac{1}{e} - 1 \right) \\
&= \frac{1}{2} \left(1 - \frac{1}{e} \right)
\end{aligned}
$$

(4) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \left(\begin{array}{l} r \geqq 0 \\ \theta : 1 \text{ 周分} \end{array} \right)$ 　とおくと

$$
\int\int_D \frac{y}{x^3} dxdy \quad \left(D : 1 \leqq x^2 + y^2 \leqq 4,\ -\sqrt{3}x \leqq y \leqq \frac{x}{\sqrt{3}} \right)
$$

$$
= \int\int_{D'} \frac{r\sin\theta}{r^3 \cos^3\theta} \cdot r dr d\theta \quad \left(D' : 1 \leqq r \leqq 2,\ -\frac{\pi}{3} \leqq \theta \leqq \frac{\pi}{6} \right)
$$

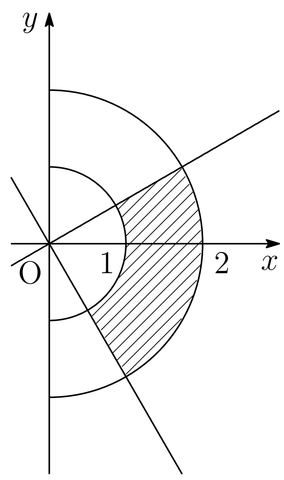
$$
= \int\int_{D'} \frac{1}{r} \cdot \frac{\sin\theta}{\cos^3\theta} dr d\theta
$$

$$
= \left(\int_1^2 \frac{1}{r} dr \right) \times \left(\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \tan\theta \cdot \frac{1}{\cos^2\theta} d\theta \right)
$$

$$
= \Big[\log r \Big]_1^2 \times \left[\frac{1}{2} \tan^2\theta \right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}}
$$

$$
= (\log 2 - 0) \times \frac{1}{2} \left(\frac{1}{3} - 3 \right)
$$

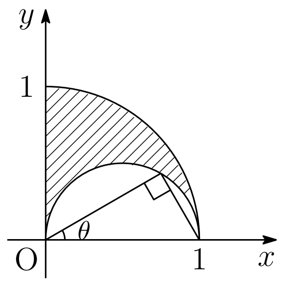
$$
= -\frac{4}{3} \log 2
$$



※ $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sin\theta}{\cos^3\theta} d\theta = -\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} (\cos\theta)^{-3} \cdot (-\sin\theta) d\theta = -\left[-\frac{1}{2} (\cos\theta)^{-2} \right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}} = \frac{1}{2} \left(\frac{4}{3} - 4 \right) = -\frac{4}{3}$

でもよい．

(5) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{pmatrix} r \geqq 0 \\ \theta : 1 \text{ 周分} \end{pmatrix}$ とおくと



$$\int\int_D x(8x-7y)dxdy \quad (D : x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0)$$

$$= \int\int_{D'} r\cos\theta(8r\cos\theta - 7r\sin\theta)\cdot rdrd\theta \quad \left(D' : 0 \leqq \theta \leqq \frac{\pi}{2},\ \cos\theta \leqq r \leqq 1\right)$$

$$= \int\int_{D'} r^3\cos\theta(8\cos\theta - 7\sin\theta)drd\theta$$

$$= \int_0^{\frac{\pi}{2}} \left\{\int_{\cos\theta}^1 r^3\cos\theta(8\cos\theta - 7\sin\theta)dr\right\}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{4}r^4\cos\theta(8\cos\theta - 7\sin\theta)\right]_{r=\cos\theta}^{r=1} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{4}(1-\cos^4\theta)\cos\theta(8\cos\theta - 7\sin\theta)d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \Big[8(\cos^2\theta - \cos^6\theta) + 7\{\cos\theta\cdot(-\sin\theta) - \cos^5\theta\cdot(-\sin\theta)\}\Big]d\theta$$

$$= \frac{1}{4}\left\{8\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{5\cdot3\cdot1}{6\cdot4\cdot2}\cdot\frac{\pi}{2}\right) + 7\left[\frac{1}{2}\cos^2\theta - \frac{1}{6}\cos^6\theta\right]_0^{\frac{\pi}{2}}\right\}$$

$$= \frac{1}{4}\left[\frac{3}{4}\pi + 7\left\{\frac{1}{2}(0-1) - \frac{1}{6}(0-1)\right\}\right]$$

$$= \frac{3}{16}\pi - \frac{7}{12}$$