科目名	微積分学B 微積分学	対象	1OB-AB	学部 研究科	理学部第一部	学科 専攻科		学籍 番号	
平成 29 年 1 月 30 日 (月) 2 回目 (～ 時限目)				担当	石川 学	学年		氏名	
試験 時間	60 分	注意 事項	① 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )						

評点

平成 28 年度後期定期試験

※解答用紙の裏面使用可

1 次の積分を求めよ．（80 点）

(1) $\displaystyle\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(4x-y)dy\right\}dx$

(2) $\displaystyle\int_{\sqrt{e}}^{e}\left(\int_{\frac{1}{x}}^{x^3}\frac{1}{xy}dy\right)dx$

(3) $\displaystyle\int_{-1}^{2}\left(\int_{-x}^{x+2}xe^{y}dy\right)dx$

(4) $\displaystyle\int_{1}^{2}\left(\int_{1}^{x}\log\frac{x}{\sqrt{y}}dy\right)dx$

(5) $\displaystyle\int_{0}^{3}\left(\int_{\frac{x}{3}}^{1}y^3e^{-y^5}dy\right)dx$ （順序変更）

(6) $\displaystyle\int\int_D\frac{y}{x^4}dxdy \quad \left(D: 1\leqq x^2+y^2\leqq 4,\ -x\leqq y\leqq\sqrt{3}\,x\right)$

(7) $\displaystyle\int\int_D y(9x-2y)dxdy \quad \left(D: x\leqq x^2+y^2\leqq 1,\ x\geqq 0,\ y\geqq 0\right)$

解答

(1)
$$\begin{aligned}\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(4x-y)dy\right\}dx &= \int_{-1}^{2}\left[4xy-\frac{1}{2}y^2\right]_{y=2x-1}^{y=x^2}dx\\ &= \int_{-1}^{2}\left[\left(4x^3-\frac{1}{2}x^4\right)-\left\{4x(2x-1)-\frac{1}{2}(2x-1)^2\right\}\right]dx\\ &= \int_{-1}^{2}\left(-\frac{1}{2}x^4+4x^3-6x^2+2x+\frac{1}{2}\right)dx\\ &= \left[-\frac{1}{10}x^5+x^4-2x^3+x^2+\frac{1}{2}x\right]_{-1}^{2}\\ &= \left(-\frac{16}{5}+16-16+4+1\right)-\left(\frac{1}{10}+1+2+1-\frac{1}{2}\right)\\ &= -\frac{9}{5}\end{aligned}$$

(2)
$$\begin{aligned}\int_{\sqrt{e}}^{e}\left(\int_{\frac{1}{x}}^{x^3}\frac{1}{xy}dy\right)dx &= \int_{\sqrt{e}}^{e}\left[\frac{\log y}{x}\right]_{y=\frac{1}{x}}^{y=x^3}dx\\ &= \int_{\sqrt{e}}^{e}\frac{3\log x-(-\log x)}{x}dx\\ &= \int_{\sqrt{e}}^{e}4\log x\cdot\frac{1}{x}dx\\ &= \left[2(\log x)^2\right]_{\sqrt{e}}^{e}\\ &= 2\left(1-\frac{1}{4}\right)\\ &= \frac{3}{2}\end{aligned}$$

(3)
$$\begin{aligned}\int_{-1}^{2}\left(\int_{-x}^{x+2}xe^y dy\right)dx &= \int_{-1}^{2}\left[xe^y\right]_{y=-x}^{y=x+2}dx\\ &= \int_{-1}^{2}(xe^{x+2}-xe^{-x})dx\\ &= \left[(xe^{x+2}-e^{x+2})-(-xe^{-x}-e^{-x})\right]_{-1}^{2}\\ &= \{2e^4-(-e)\}-(e^4-e)+\{2e^{-2}-(-e)\}+(e^{-2}-e)\\ &= e^4+2e+\frac{3}{e^2}\end{aligned}$$

(4)
$$\begin{aligned}\int_{1}^{2}\left(\int_{1}^{x}\log\frac{x}{\sqrt{y}}dy\right)dx &= \int_{1}^{2}\left\{\int_{1}^{x}\left(\log x-\frac{1}{2}\log y\right)dy\right\}dx\\ &= \int_{1}^{2}\left[y\log x-\frac{1}{2}(y\log y-y)\right]_{y=1}^{y=x}dx\\ &= \int_{1}^{2}\left\{(x-1)\log x-\frac{1}{2}(x\log x-1\cdot 0)+\frac{1}{2}(x-1)\right\}dx\\ &= \int_{1}^{2}\left\{\frac{1}{2}x\log x-\log x+\frac{1}{2}(x-1)\right\}dx\\ &= \left[\frac{1}{2}\left(\frac{1}{2}x^2\log x-\frac{1}{4}x^2\right)-(x\log x-x)+\frac{1}{4}(x-1)^2\right]_{1}^{2}\\ &= \frac{1}{4}(4\log 2-1\cdot 0)-\frac{1}{8}(4-1)-(2\log 2-1\cdot 0)+(2-1)+\frac{1}{4}(1-0)\\ &= \frac{7}{8}-\log 2\end{aligned}$$

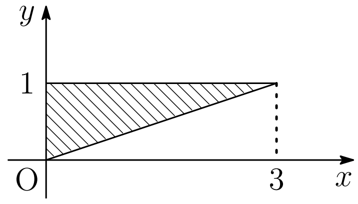
(5) 積分領域は

$D : 0 \leqq x \leqq 3,\ \dfrac{x}{3} \leqq y \leqq 1$

であるが，これは

$D : 0 \leqq y \leqq 1,\ 0 \leqq x \leqq 3y$

でもあるから



$$\begin{aligned}
\int_0^3 \left(\int_{\frac{x}{3}}^1 y^3 e^{-y^5} dy \right) dx &= \int_0^1 \left(\int_0^{3y} y^3 e^{-y^5} dx \right) dy \\
&= \int_0^1 \left[x y^3 e^{-y^5} \right]_{x=0}^{x=3y} dy \\
&= \int_0^1 3y^4 e^{-y^5} dy \\
&= -\frac{3}{5} \int_0^1 e^{-y^5} \cdot (-5y^4) dy \\
&= -\frac{3}{5} \left[e^{-y^5} \right]_0^1 \\
&= -\frac{3}{5} \left(\frac{1}{e} - 1 \right) \\
&= \frac{3}{5} \left(1 - \frac{1}{e} \right)
\end{aligned}$$

(6) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{pmatrix} r \geqq 0 \\ \theta : 1 \text{ 周分} \end{pmatrix}$ とおくと

$\displaystyle\int\int_D \frac{y}{x^4} dxdy \quad \left(D : 1 \leqq x^2 + y^2 \leqq 4,\ -x \leqq y \leqq \sqrt{3}x \right)$

$\displaystyle= \int\int_{D'} \frac{r\sin\theta}{r^4 \cos^4\theta} \cdot r dr d\theta \quad \left(D' : 1 \leqq r \leqq 2,\ -\frac{\pi}{4} \leqq \theta \leqq \frac{\pi}{3} \right)$

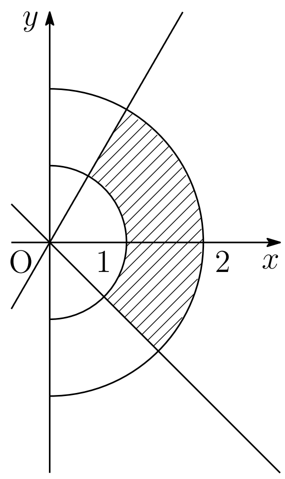
$\displaystyle= \int\int_{D'} \frac{1}{r^2} \cdot \frac{\sin\theta}{\cos^4\theta} dr d\theta$

$\displaystyle= \left(\int_1^2 \frac{1}{r^2} dr \right) \times \left\{ -\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} (\cos\theta)^{-4} \cdot (-\sin\theta) d\theta \right\}$

$\displaystyle= \left[-\frac{1}{r} \right]_1^2 \times \left\{ -\left[-\frac{1}{3} (\cos\theta)^{-3} \right]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \right\}$

$\displaystyle= -\left(\frac{1}{2} - 1 \right) \times \frac{1}{3} (8 - 2\sqrt{2})$

$\displaystyle= \frac{4}{3} - \frac{\sqrt{2}}{3}$



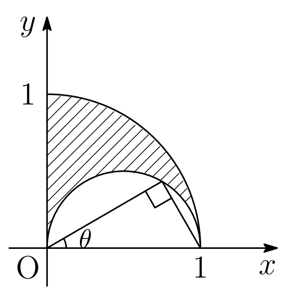
(7) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{pmatrix} r \geqq 0 \\ \theta : 1 \text{ 周分} \end{pmatrix}$ とおくと

$\displaystyle\int\int_D y(9x - 2y) dxdy \quad \left(D : x \leqq x^2 + y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0 \right)$

$\displaystyle= \int\int_{D'} r\sin\theta (9r\cos\theta - 2r\sin\theta) \cdot r dr d\theta \quad \left(D' : 0 \leqq \theta \leqq \frac{\pi}{2},\ \cos\theta \leqq r \leqq 1 \right)$

$\displaystyle= \int\int_{D'} r^3 \sin\theta (9\cos\theta - 2\sin\theta) dr d\theta$

$\displaystyle= \int_0^{\frac{\pi}{2}} \left\{ \int_{\cos\theta}^1 r^3 \sin\theta (9\cos\theta - 2\sin\theta) dr \right\} d\theta$



$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{4} r^4 \sin\theta (9\cos\theta - 2\sin\theta)\right]_{r=\cos\theta}^{r=1} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{4}(1-\cos^4\theta)\sin\theta(9\cos\theta - 2\sin\theta)d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \{9(\cos\theta\sin\theta - \cos^5\theta\sin\theta) - 2(\sin^2\theta - \cos^4\theta\sin^2\theta)\}d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \Big[9\{-\cos\theta\cdot(-\sin\theta) + \cos^5\theta\cdot(-\sin\theta)\} - 2(\sin^2\theta - \cos^4\theta + \cos^6\theta)\Big]d\theta$$

$$= \frac{1}{4}\left\{9\left[-\frac{1}{2}\cos^2\theta + \frac{1}{6}\cos^6\theta\right]_0^{\frac{\pi}{2}} - 2\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} + \frac{5\cdot 3\cdot 1}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}\right)\right\}$$

$$= \frac{1}{4}\left[9\left\{-\frac{1}{2}(0-1) + \frac{1}{6}(0-1)\right\} - \frac{7}{16}\pi\right]$$

$$= \frac{3}{4} - \frac{7}{64}\pi$$