科目名	微積分学A 微積分学	対象	1OB-AB	学部 研究科	理学部第一部	学科 専攻科		学籍 番号		評点
平成 28 年 8 月 1 日 (月) 3 回目 (～ 時限目)				担当	石川 学	学年		氏名		
試験 時間	60 分	注意 事項	1. 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )							

平成 28 年度前期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ (1) 次の等式が成り立つような定数 A, B, C の値を求めよ．**(1)** は答えのみでよい．

$$\frac{x^2-x-19}{(x-4)(x^2-10x+31)} = \frac{A}{x-4} + \frac{Bx+C}{x^2-10x+31}$$

(2) $\displaystyle\int \frac{x^2-x-19}{(x-4)(x^2-10x+31)}dx$ を求めよ．

$\boxed{2}$ $\tan\dfrac{x}{2} = t$ とおくことにより，$\displaystyle\int \frac{\sin x + 2\cos x + 6}{(\sin x + \cos x + 3)\sin x}dx$ を求めよ．

$\boxed{3}$ $\sqrt{4x^2+3x+6} + 2x = t$ とおくことにより，$\displaystyle\int \frac{1}{(x+2)\sqrt{4x^2+3x+6}}dx$ を求めよ．

$\boxed{4}$ 次を求めよ．

(1) $\displaystyle\int_{2-\sqrt{3}}^{2+\sqrt{3}} \arctan x\, dx$　　(2) $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{(\arcsin x)^5\sqrt{1-x^2}}dx$

$\boxed{5}$ 次を求めよ．

(1) $\displaystyle\int_0^e \frac{(\log x)^2}{\sqrt{x}}dx$　　(2) $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{\infty} \frac{\arctan x}{x^4}dx$

東京理科大学　　年　月　日　**試験答案**　　評点

科目名		担当	先生
所属 学籍番号	学部　学科　番	氏名	

1

(1) $\dfrac{x^2-x-19}{(x-4)(x^2-10x+31)} = \dfrac{A}{x-4} + \dfrac{Bx+C}{x^2-10x+31}$

の分母をはらうと

$x^2-x-19 = A(x^2-10x+31)+(Bx+C)(x-4)$

・$x=4$ 代入　$-7=7A$　$\therefore A=-1$

・$x=0$ 代入　$-19=31A-4C$

$-19=-31-4C$　$\therefore C=-3$

・x^2の係数比較　$1=A+B$

$1=-1+B$　$\therefore B=2$

よって　$(A, B, C)=(-1, 2, -3)$

(2) $\displaystyle\int \frac{x^2-x-19}{(x-4)(x^2-10x+31)}dx$

$\displaystyle= \int \left(-\frac{1}{x-4} + \frac{2x-3}{x^2-10x+31}\right)dx$

$\displaystyle= \int \left\{-\frac{1}{x-4} + \frac{(2x-10)+7}{x^2-10x+31}\right\}dx$

$\displaystyle= \int \left\{-\frac{1}{x-4} + \frac{2x-10}{x^2-10x+31} + \frac{7}{(\sqrt{6})^2+(x-5)^2}\right\}dx$

$\displaystyle= -\log|x-4| + \log(x^2-10x+31) + \frac{7}{\sqrt{6}}\arctan\frac{x-5}{\sqrt{6}}$

2

$\tan\dfrac{x}{2}=t$ とおくと

$\displaystyle\int \frac{\sin x+2\cos x+6}{(\sin x+\cos x+3)\sin x}dx$

$\displaystyle= \int \frac{\frac{2t}{1+t^2}+2\cdot\frac{1-t^2}{1+t^2}+6}{\left(\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}+3\right)\frac{2t}{1+t^2}}\cdot\frac{2}{1+t^2}dt$

$\displaystyle= \int \frac{2t^2+t+4}{t(t^2+t+2)}dt$

$\displaystyle= \int \frac{2(t^2+t+2)-t}{t(t^2+t+2)}dt$

$\displaystyle= \int \left(\frac{2}{t} - \frac{1}{t^2+t+2}\right)dt$

$\displaystyle= \int \left\{\frac{2}{t} - \frac{1}{(\frac{\sqrt{7}}{2})^2+(t+\frac{1}{2})^2}\right\}dt$

$\displaystyle= 2\log|t| - \frac{1}{\frac{\sqrt{7}}{2}}\arctan\frac{t+\frac{1}{2}}{\frac{\sqrt{7}}{2}}$

$\displaystyle= 2\log|t| - \frac{2}{\sqrt{7}}\arctan\frac{2t+1}{\sqrt{7}}$

3

$\sqrt{4x^2+3x+6}+2x=t$ とおくと $\sqrt{4x^2+3x+6}=t-2x$

両辺を2乗すると

$4x^2+3x+6=t^2-4tx+4x^2$

$(4t+3)x=t^2-6$

$\therefore x=\dfrac{t^2-6}{4t+3}$

また

$\dfrac{dx}{dt}=\dfrac{2t\cdot(4t+3)-(t^2-6)\cdot4}{(4t+3)^2}=\dfrac{2(2t^2+3t+12)}{(4t+3)^2}$

さらに

$\sqrt{4x^2+3x+6}=t-2x=t-2\cdot\dfrac{t^2-6}{4t+3}=\dfrac{2t^2+3t+12}{4t+3}$

よって

$\displaystyle\int \frac{1}{(x+2)\sqrt{4x^2+3x+6}}dx$

$\displaystyle= \int \frac{1}{\left(\frac{t^2-6}{4t+3}+2\right)\frac{2t^2+3t+12}{4t+3}}\cdot\frac{2(2t^2+3t+12)}{(4t+3)^2}dt$

$\displaystyle= \int \frac{2}{t^2+8t}dt$

$\displaystyle= \int \frac{2}{t(t+8)}dt$

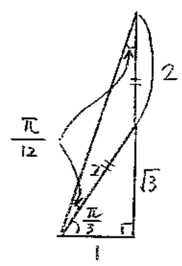
$\displaystyle= \int \frac{1}{4}\left(\frac{1}{t}-\frac{1}{t+8}\right)dt$

$\displaystyle= \frac{1}{4}\left(\log|t|-\log|t+8|\right)$

4

(1) $\arctan(2+\sqrt{3})=\dfrac{\pi}{3}+\dfrac{\pi}{12}=\dfrac{5}{12}\pi$

$\arctan(2-\sqrt{3})=\arctan\dfrac{1}{2+\sqrt{3}}=\dfrac{\pi}{12}$



だから

$\displaystyle\int_{2-\sqrt{3}}^{2+\sqrt{3}} \arctan x\,dx$

$\displaystyle= \left[x\arctan x - \frac{1}{2}\log(1+x^2)\right]_{2-\sqrt{3}}^{2+\sqrt{3}}$

$\displaystyle= \left\{(2+\sqrt{3})\cdot\frac{5}{12}\pi - (2-\sqrt{3})\cdot\frac{\pi}{12}\right\} - \frac{1}{2}\left\{\log(8+4\sqrt{3})-\log(8-4\sqrt{3})\right\}$

$\displaystyle= \frac{4+3\sqrt{3}}{6}\pi - \frac{1}{2}\log\frac{8+4\sqrt{3}}{8-4\sqrt{3}}$

$\displaystyle= \frac{4+3\sqrt{3}}{6}\pi - \frac{1}{2}\log\frac{2+\sqrt{3}}{2-\sqrt{3}}$

$\displaystyle= \frac{4+3\sqrt{3}}{6}\pi - \frac{1}{2}\log\frac{(2+\sqrt{3})^2}{4-3}$

$\displaystyle= \frac{4+3\sqrt{3}}{6}\pi - \log(2+\sqrt{3})$

(2) $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{(\arcsin x)^5\sqrt{1-x^2}}\,dx$

$$= \int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} (\arcsin x)^{-5}\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \left[-\frac{1}{4}(\arcsin x)^{-4}\right]_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}}$$

$$= -\frac{1}{4}\left\{\left(\frac{3}{\pi}\right)^4-\left(\frac{4}{\pi}\right)^4\right\}$$

$$= \frac{1}{4}\cdot\frac{256-81}{\pi^4}$$

$$= \frac{175}{4\pi^4}$$

$\boxed{5}$

(1) $\displaystyle\int \frac{(\log x)^2}{\sqrt{x}}\,dx$

$$\begin{array}{cc} (\log x)^2 & 2\sqrt{x} \\ 2\log x\cdot\frac{1}{x} & \frac{1}{\sqrt{x}} \end{array} \qquad \begin{array}{cc} \log x & 2\sqrt{x} \\ \frac{1}{x} & \frac{1}{\sqrt{x}} \end{array}$$

$$= 2\sqrt{x}(\log x)^2 - 4\int\frac{\log x}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}(\log x)^2 - 4\left(2\sqrt{x}\log x - 2\int\frac{1}{\sqrt{x}}\,dx\right)$$

$$= 2\sqrt{x}(\log x)^2 - 4\left(2\sqrt{x}\log x - 4\sqrt{x}\right)$$

$$= 2\sqrt{x}(\log x)^2 - 8\sqrt{x}\log x + 16\sqrt{x}$$

だから，$0<\varepsilon<e$ に対して

$$\int_{\varepsilon}^{e}\frac{(\log x)^2}{\sqrt{x}}\,dx = \left[2\sqrt{x}(\log x)^2 - 8\sqrt{x}\log x + 16\sqrt{x}\right]_{\varepsilon}^{e}$$

$$= \left(2\sqrt{e}\cdot 1 - 8\sqrt{e}\cdot 1 + 16\sqrt{e}\right)$$

$$-\left\{2\sqrt{\varepsilon}(\log\varepsilon)^2 - 8\sqrt{\varepsilon}\log\varepsilon + 16\sqrt{\varepsilon}\right\}$$

よって

$$\int_0^e\frac{(\log x)^2}{\sqrt{x}}\,dx = \lim_{\varepsilon\to+0}\int_{\varepsilon}^{e}\frac{(\log x)^2}{\sqrt{x}}\,dx$$

$$= \lim_{\varepsilon\to+0}\left\{10\sqrt{e} - 2\sqrt{\varepsilon}(\log\varepsilon)^2 + 8\sqrt{\varepsilon}\log\varepsilon - 16\sqrt{\varepsilon}\right\}$$

$$= 10\sqrt{e}$$

(2) $\displaystyle\int\frac{\arctan x}{x^4}\,dx = -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\frac{1}{x^3(1+x^2)}\,dx$

$$\begin{array}{cc} \arctan x & -\frac{1}{3x^3} \\ \frac{1}{1+x^2} & \frac{1}{x^4} \end{array}$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\frac{(1+x^2)-x^2}{x^3(1+x^2)}\,dx$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\left\{\frac{1}{x^3} - \frac{1}{x(1+x^2)}\right\}dx$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\left\{\frac{1}{x^3} - \frac{(1+x^2)-x^2}{x(1+x^2)}\right\}dx$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\left\{\frac{1}{x^3} - \left(\frac{1}{x} - \frac{x}{1+x^2}\right)\right\}dx$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\int\left(\frac{1}{x^3} + \frac{1}{2}\cdot\frac{2x}{1+x^2} - \frac{1}{x}\right)dx$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\left\{-\frac{1}{2x^2} + \frac{1}{2}\log(1+x^2) - \log|x|\right\}$$

$$= -\frac{\arctan x}{3x^3} + \frac{1}{3}\left\{-\frac{1}{2x^2} + \frac{1}{2}\log(1+x^2) - \frac{1}{2}\log x^2\right\}$$

$$= -\frac{\arctan x}{3x^3} - \frac{1}{6x^2} + \frac{1}{6}\log\frac{1+x^2}{x^2}$$

だから，$R>\frac{1}{\sqrt{3}}$ に対して

$$\int_{\frac{1}{\sqrt{3}}}^{R}\frac{\arctan x}{x^4}\,dx = \left[-\frac{\arctan x}{3x^3} - \frac{1}{6x^2} + \frac{1}{6}\log\frac{1+x^2}{x^2}\right]_{\frac{1}{\sqrt{3}}}^{R}$$

$$= \left(-\frac{\arctan R}{3R^3} - \frac{1}{6R^2} + \frac{1}{6}\log\frac{1+R^2}{R^2}\right)$$

$$-\left(-\frac{\frac{\pi}{6}}{3\cdot\frac{1}{3\sqrt{3}}} - \frac{1}{6\cdot\frac{1}{3}} + \frac{1}{6}\log\frac{1+\frac{1}{3}}{\frac{1}{3}}\right)$$

よって

$$\int_{\frac{1}{\sqrt{3}}}^{\infty}\frac{\arctan x}{x^4}\,dx$$

$$= \lim_{R\to\infty}\int_{\frac{1}{\sqrt{3}}}^{R}\frac{\arctan x}{x^4}\,dx$$

$$= \lim_{R\to\infty}\left\{\frac{\sqrt{3}}{6}\pi + \frac{1}{2} - \frac{1}{3}\log 2 - \frac{\arctan R}{3R^3} - \frac{1}{6R^2} + \frac{1}{6}\log\left(\frac{1}{R^2}+1\right)\right\}$$

$$= \frac{\sqrt{3}}{6}\pi + \frac{1}{2} - \frac{1}{3}\log 2$$