科目名	解析学	対象	2OB	学部 研究科	理学部第一部	学科 専攻科		学籍 番号		評点
平成 26 年 7 月 24 日 (木) 3 回目 (～ 時限目)				担当	石川 学	学年		氏名		
試験 時間	60 分	注意 事項	1. 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )							

平成 26 年度前期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ $(-1+\sqrt{3}i)^{1+\frac{i}{\pi}}$ を $a+bi$ $(a, b \in \mathbb{R})$ または $r(\cos\theta + i\sin\theta)$ $(r>0,\ \theta \in \mathbb{R})$ の形にせよ.

(10 点)

$\boxed{2}$ 次の複素積分の値を求めよ. (60 点)

(1) $\displaystyle\int_C (\overline{z}+1)dz \qquad C : z = t^2 + it \quad (-1 \leqq t \leqq 3)$

(2) $\displaystyle\int_C \overline{z}\,\mathrm{Re} z dz \qquad C : z = t + \frac{i}{t} \quad \left(\frac{1}{2} \leqq t \leqq 2\right)$

(3) $\displaystyle\int_C \mathrm{Im} z dz \qquad C : z = t + i\log t \quad (e \leqq t \leqq e^2)$

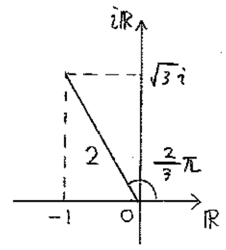
(4) $\displaystyle\int_C |z|^2 dz \qquad C : z = t + i\sin t \quad \left(0 \leqq t \leqq \frac{\pi}{2}\right)$

(5) $\displaystyle\int_C \mathrm{Log} z dz \qquad C : z = 1 + it \quad (0 \leqq t \leqq 1)$

東京理科大学　　年　月　日　**試験答案**　　評点

科目名		担当	先生
所属 学籍番号	学部　学科　番	氏名	H26 (2014) 前定

1



$$\left(1+\frac{i}{\pi}\right)\log(-1+\sqrt{3}i) = \left(1+\frac{i}{\pi}\right)\left\{\log 2 + i\left(\frac{2}{3}\pi + 2m\pi\right)\right\}$$
$$= \log 2 - \frac{2}{3} - 2m + i\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi + 2m\pi\right)$$

であるから

$$(-1+\sqrt{3}i)^{1+\frac{i}{\pi}} = e^{\log 2 - \frac{2}{3} - 2m + i\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi + 2m\pi\right)}$$
$$= e^{\log 2 - \frac{2}{3} - 2m}\left\{\cos\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi + 2m\pi\right) + i\sin\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi + 2m\pi\right)\right\}$$
$$= 2e^{-\frac{2}{3}-2m}\left\{\cos\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi\right) + i\sin\left(\frac{1}{\pi}\log 2 + \frac{2}{3}\pi\right)\right\} \quad (m \in \mathbb{Z})$$

2

(1) $\displaystyle\int_C (\bar{z}+1)\,dz \qquad C: z = t^2 + it \quad (-1 \leqq t \leqq 3)$

$$= \int_{-1}^{3}(t^2 - it + 1)\cdot(2t+i)\,dt = \int_{-1}^{3}\{2t^3 + 3t + i(-t^2+1)\}dt = \left[\frac{1}{2}t^4 + \frac{3}{2}t^2 + i\left(-\frac{1}{3}t^3 + t\right)\right]_{-1}^{3}$$
$$= \frac{1}{2}(81-1) + \frac{3}{2}(9-1) + i\left[-\frac{1}{3}\{27-(-1)\} + \{3-(-1)\}\right] = 52 - \frac{16}{3}i$$

(2) $\displaystyle\int_C \bar{z}\,\mathrm{Re}\,z\,dz \qquad C: z = t + \frac{i}{t} \quad \left(\frac{1}{2} \leqq t \leqq 2\right)$

$$= \int_{\frac{1}{2}}^{2}\left(t - \frac{i}{t}\right)t\cdot\left(1 - \frac{i}{t^2}\right)dt = \int_{\frac{1}{2}}^{2}\left(t^2 - \frac{1}{t^2} - 2i\right)dt = \left[\frac{1}{3}t^3 + \frac{1}{t} - 2it\right]_{\frac{1}{2}}^{2}$$
$$= \frac{1}{3}\left(8 - \frac{1}{8}\right) + \left(\frac{1}{2} - 2\right) - 2i\left(2 - \frac{1}{2}\right) = \frac{9}{8} - 3i$$

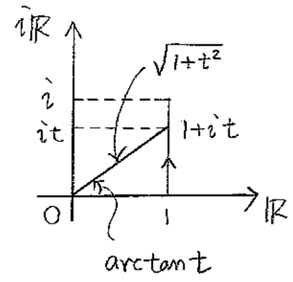
(3) $\displaystyle\int_C \mathrm{Im}\,z\,dz \qquad C: z = t + i\log t \quad (e \leqq t \leqq e^2)$

$$= \int_{e}^{e^2}\log t\cdot\left(1 + \frac{i}{t}\right)dt = \int_{e}^{e^2}\left(\log t + i\log t\cdot\frac{1}{t}\right)dt = \left[t\log t - t + \frac{i}{2}(\log t)^2\right]_{e}^{e^2}$$
$$= (e^2\cdot 2 - e\cdot 1) - (e^2 - e) + \frac{i}{2}(4-1) = e^2 + \frac{3}{2}i$$

(4) $\displaystyle\int_C |z|^2\,dz \qquad C: z = t + i\sin t \quad \left(0 \leqq t \leqq \frac{\pi}{2}\right)$

$$= \int_0^{\frac{\pi}{2}}(t^2 + \sin^2 t)\cdot(1 + i\cos t)\,dt = \int_0^{\frac{\pi}{2}}\{t^2 + \sin^2 t + i(t^2\cos t + \sin^2 t\cdot\cos t)\}dt$$
$$= \left[\frac{1}{3}t^3\right]_0^{\frac{\pi}{2}} + \frac{1}{2}\cdot\frac{\pi}{2} + i\left[(t^2\sin t + 2t\cos t - 2\sin t) + \frac{1}{3}\sin^3 t\right]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi^3}{24} + \frac{\pi}{4} + i\left\{\left(\frac{\pi^2}{4}\cdot 1 - 0\cdot 0\right) + 2\left(\frac{\pi}{2}\cdot 0 - 0\cdot 1\right) - 2(1-0) + \frac{1}{3}(1-0)\right\}$$
$$= \frac{\pi^3}{24} + \frac{\pi}{4} + \left(\frac{\pi^2}{4} - \frac{5}{3}\right)i$$

(5) $\displaystyle\int_C \mathrm{Log}\, z\, dz \qquad C: z = 1 + it \quad (0 \leqq t \leqq 1)$



$\displaystyle = \int_0^1 \mathrm{Log}(1+it)\cdot i\,dt$

主値（偏角：$-\pi \sim \pi$）

$\displaystyle = \int_0^1 \left(\log\sqrt{1+t^2} + i\arctan t\right)\cdot i\,dt$

$\displaystyle = \int_0^1 \left\{-\arctan t + \frac{i}{2}\log(1+t^2)\right\} dt$

$\begin{array}{cc} \arctan t & t \\ \frac{1}{1+t^2} & 1 \end{array} \qquad \begin{array}{cc} \log(1+t^2) & t \\ \frac{2t}{1+t^2} & 1 \end{array}$

ここで

$$\int_0^1 \arctan t\, dt = \Big[t\arctan t\Big]_0^1 - \int_0^1 \frac{t}{1+t^2}dt = \left(1\cdot\frac{\pi}{4} - 0\cdot 0\right) - \frac{1}{2}\int_0^1 \frac{2t}{1+t^2}dt$$

$$= \frac{\pi}{4} - \frac{1}{2}\Big[\log(1+t^2)\Big]_0^1 = \frac{\pi}{4} - \frac{1}{2}(\log 2 - 0) = \frac{\pi}{4} - \frac{1}{2}\log 2$$

$$\int_0^1 \log(1+t^2)dt = \Big[t\log(1+t^2)\Big]_0^1 - \int_0^1 \frac{2t^2}{1+t^2}dt = (1\cdot\log 2 - 0\cdot 0) - 2\int_0^1 \frac{(1+t^2)-1}{1+t^2}dt$$

$$= \log 2 - 2\int_0^1 \left(1 - \frac{1}{1+t^2}\right)dt = \log 2 - 2\Big[t - \arctan t\Big]_0^1$$

$$= \log 2 - 2\left\{1 - \left(\frac{\pi}{4} - 0\right)\right\} = \log 2 - 2 + \frac{\pi}{2}$$

であるから

$$\int_C \mathrm{Log}\, z\, dz = \frac{1}{2}\log 2 - \frac{\pi}{4} + \left(\frac{1}{2}\log 2 + \frac{\pi}{4} - 1\right) i$$