科目名	解析学	対象	2OB	学部研究科	理学部第一部	学科専攻科		学籍番号	
平成 27 年 7 月 30 日 (木) 3 回目 (～ 時限目)				担当	石川 学	学年		氏名	
試験時間	60 分	注意事項	1. 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )						

評 点

平成 27 年度前期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ $(\sqrt{3}+i)^{1-\frac{i}{\pi}}$ を $a+bi$ $(a, b \in \mathbb{R})$ または $r(\cos\theta + i\sin\theta)$ $(r>0,\ \theta \in \mathbb{R})$ の形にせよ．

(10 点)

$\boxed{2}$ 次の複素積分の値を求めよ． (60 点)

(1) $\displaystyle\int_C (\overline{z}+1)dz$ $\quad C: z = t^2 - it \quad (-2 \leqq t \leqq 3)$

(2) $\displaystyle\int_C \frac{\overline{z}}{\mathrm{Im}z}dz$ $\quad C: z = \dfrac{1}{t} + it \quad \left(\dfrac{1}{2} \leqq t \leqq 2\right)$

(3) $\displaystyle\int_C \mathrm{Re}z dz$ $\quad C: z = \arctan t + 2it \quad (0 \leqq t \leqq 1)$

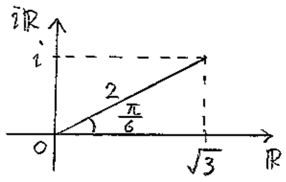
(4) $\displaystyle\int_C |z|^2 dz$ $\quad C: z = t - i\cos t \quad \left(0 \leqq t \leqq \dfrac{\pi}{2}\right)$

(5) $\displaystyle\int_C \mathrm{Log}\overline{z} dz$ $\quad C: z = t - it \quad (1 \leqq t \leqq 2)$

東京理科大学　　年　月　日　**試験答案**　　評点

科目名		担当	先生
所属 学籍番号	学部　学科　番	氏名	

1



$$\left(1-\frac{i}{\pi}\right)\log(\sqrt{3}+i) = \left(1-\frac{i}{\pi}\right)\left\{\log 2 + i\left(\frac{\pi}{6}+2n\pi\right)\right\}$$

$$= \log 2 + \frac{1}{6} + 2n + i\left(-\frac{1}{\pi}\log 2 + \frac{\pi}{6} + 2n\pi\right) \qquad (n \in \mathbb{Z})$$

だから

$$(\sqrt{3}+i)^{1-\frac{i}{\pi}} = e^{\left(1-\frac{i}{\pi}\right)\log(\sqrt{3}+i)}$$

$$= e^{\log 2 + \frac{1}{6} + 2n + i\left(-\frac{1}{\pi}\log 2 + \frac{\pi}{6} + 2n\pi\right)}$$

$$= 2e^{\frac{1}{6}+2n}\left\{\cos\left(-\frac{1}{\pi}\log 2 + \frac{\pi}{6}\right) + i\sin\left(-\frac{1}{\pi}\log 2 + \frac{\pi}{6}\right)\right\} \qquad (n \in \mathbb{Z})$$

2

(1) $C : z = t^2 - it \quad (-2 \leqq t \leqq 3) \longrightarrow \frac{dz}{dt} = 2t - i$

$$\int_C (\overline{z}+1)\,dz = \int_{-2}^{3} (t^2+it+1)\cdot(2t-i)\,dt = \int_{-2}^{3} \left\{2t^3+3t+i(t^2-1)\right\}dt$$

$$= \left[\frac{1}{2}t^4 + \frac{3}{2}t^2 + i\left(\frac{1}{3}t^3 - t\right)\right]_{-2}^{3} = \frac{1}{2}(81-16) + \frac{3}{2}(9-4) + i\left[\frac{1}{3}\{27-(-8)\} - \{3-(-2)\}\right]$$

$$= 40 + \frac{20}{3}i$$

(2) $C : z = \frac{1}{t} + it \quad \left(\frac{1}{2} \leqq t \leqq 2\right) \longrightarrow \frac{dz}{dt} = -\frac{1}{t^2} + i$

$$\int_C \frac{\overline{z}}{\operatorname{Im} z}\,dz = \int_{\frac{1}{2}}^{2} \frac{\frac{1}{t} - it}{t}\cdot\left(-\frac{1}{t^2}+i\right)dt = \int_{\frac{1}{2}}^{2}\left(-\frac{1}{t^4} + 1 + \frac{2i}{t^2}\right)dt = \left[\frac{1}{3t^3} + t - \frac{2i}{t}\right]_{\frac{1}{2}}^{2}$$

$$= \frac{1}{3}\left(\frac{1}{8} - 8\right) + \left(2 - \frac{1}{2}\right) - 2i\left(\frac{1}{2} - 2\right) = -\frac{9}{8} + 3i$$

(3) $C : z = \arctan t + 2it \quad (0 \leqq t \leqq 1) \longrightarrow \frac{dz}{dt} = \frac{1}{1+t^2} + 2i$

$$\int_C \operatorname{Re} z\,dz = \int_0^1 \arctan t\cdot\left(\frac{1}{1+t^2} + 2i\right)dt = \int_0^1 \left(\arctan t\cdot\frac{1}{1+t^2} + 2i\arctan t\right)dt$$

$$= \left[\frac{1}{2}(\arctan t)^2 + 2i\left\{t\arctan t - \frac{1}{2}\log(1+t^2)\right\}\right]_0^1$$

$$= \frac{1}{2}\left(\frac{\pi^2}{16} - 0\right) + 2i\left\{\left(1\cdot\frac{\pi}{4} - 0\cdot 0\right) - \frac{1}{2}(\log 2 - 0)\right\} = \frac{\pi^2}{32} + \left(\frac{\pi}{2} - \log 2\right)i$$

(4) $C : z = t - i\cos t \quad (0 \leqq t \leqq \frac{\pi}{2}) \quad \longrightarrow \quad \frac{dz}{dt} = 1 + i\sin t$

$$\int_C |z|^2 dz = \int_0^{\frac{\pi}{2}} (t^2 + \cos^2 t)\cdot(1 + i\sin t)\,dt = \int_0^{\frac{\pi}{2}} \{t^2 + \cos^2 t + i(t^2 \sin t + \cos^2 t \sin t)\}\,dt$$

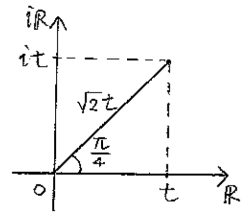
$$= \int_0^{\frac{\pi}{2}} \left[t^2 + \underset{\text{公式}}{\underline{\cos^2 t}} + i\,\{t^2 \sin t - \cos^2 t\cdot(-\sin t)\}\right] dt$$

$$= \left[\frac{1}{3}t^3\right]_0^{\frac{\pi}{2}} + \frac{1}{2}\cdot\frac{\pi}{2} + i\left[(-t^2\cos t + 2t\sin t + 2\cos t) - \frac{1}{3}\cos^3 t\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{3}\cdot\frac{\pi^3}{8} + \frac{\pi}{4} + i\left\{-\left(\frac{\pi^2}{4}\cdot 0 - 0\cdot 1\right) + 2\left(\frac{\pi}{2}\cdot 1 - 0\cdot 0\right) + 2(0-1) - \frac{1}{3}(0-1)\right\}$$

$$= \frac{\pi^3}{24} + \frac{\pi}{4} + \left(\pi - \frac{5}{3}\right)i$$

(5) $C : z = t - it \quad (1 \leqq t \leqq 2) \quad \longrightarrow \quad \frac{dz}{dt} = 1 - i$



$$\int_C \mathrm{Log}\,\bar{z}\,dz = \int_1^2 \mathrm{Log}(t + it)\cdot(1-i)\,dt$$

$$= \int_1^2 \left(\log\sqrt{2}t + \frac{\pi}{4}i\right)\cdot(1-i)\,dt$$

$$= (1-i)\int_1^2 \left(\log t + \frac{1}{2}\log 2 + \frac{\pi}{4}i\right) dt$$

$$= (1-i)\left[(t\log t - t) + \left(\frac{1}{2}\log 2 + \frac{\pi}{4}i\right)t\right]_1^2$$

$$= (1-i)\left\{(2\log 2 - 1\cdot 0) - (2-1) + \left(\frac{1}{2}\log 2 + \frac{\pi}{4}i\right)(2-1)\right\}$$

$$= (1-i)\left(\frac{5}{2}\log 2 - 1 + \frac{\pi}{4}i\right)$$

$$= \frac{5}{2}\log 2 - 1 + \frac{\pi}{4} + \left(-\frac{5}{2}\log 2 + 1 + \frac{\pi}{4}\right)i$$