

第1章 § 2 補助プリント

$$(1) \forall x p(x) \implies \exists x p(x)$$

$x : x_1, x_2, x_3$ のとき

$$\forall x p(x) \iff p(x_1) \& p(x_2) \& p(x_3)$$

$$\exists x p(x) \iff p(x_1) \text{ or } p(x_2) \text{ or } p(x_3)$$

$p(x_1)$	$p(x_2)$	$p(x_3)$	$\forall x p(x) \implies \exists x p(x)$		
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	F

$$(2) (\forall x p(x)) \text{ or } (\forall x q(x)) \implies \forall x [p(x) \text{ or } q(x)]$$

$x : x_1, x_2$ のとき

$$(\forall x p(x)) \text{ or } (\forall x q(x)) \iff (p(x_1) \& p(x_2)) \text{ or } (q(x_1) \& q(x_2))$$

$$\forall x [p(x) \text{ or } q(x)] \iff (p(x_1) \text{ or } q(x_1)) \& (p(x_2) \text{ or } q(x_2))$$

$p(x_1)$	$p(x_2)$	$q(x_1)$	$q(x_2)$	$(\forall x p(x)) \text{ or } (\forall x q(x)) \implies \forall x [p(x) \text{ or } q(x)]$		
T	T	T	T	T	T	T
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	F
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	F
F	F	F	F	F	T	F

$$(3) \forall x[p(x) \Rightarrow q(x)] \implies (\forall x p(x)) \Rightarrow (\forall x q(x))$$

$x : x_1, x_2$ のとき

$$\forall x[p(x) \Rightarrow q(x)] \iff (p(x_1) \Rightarrow q(x_1)) \& (p(x_2) \Rightarrow q(x_2))$$

$$(\forall x p(x)) \Rightarrow (\forall x q(x)) \iff (p(x_1) \& p(x_2)) \Rightarrow (q(x_1) \& q(x_2))$$

$p(x_1)$	$p(x_2)$	$q(x_1)$	$q(x_2)$	$\forall x[p(x) \Rightarrow q(x)]$	$\implies$	$(\forall x p(x)) \Rightarrow (\forall x q(x))$
T	T	T	T	T	T	T
T	T	T	F	F	T	F
T	T	F	T	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	T	F	T	T	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	F	F	T	T
F	T	F	T	T	T	T
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	T	F	T	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

$$(4) \exists x \forall y p(x, y) \implies \forall y \exists x p(x, y)$$

$x : x_1, x_2, \quad y : y_1, y_2$ のとき

$$\exists x \forall y p(x, y) \iff (\forall y p(x_1, y)) \text{ or } (\forall y p(x_2, y))$$

$$\forall y \exists x p(x, y) \iff (\exists x p(x, y_1)) \& (\exists x p(x, y_2))$$

$p(x_1, y_1)$	$p(x_1, y_2)$	$p(x_2, y_1)$	$p(x_2, y_2)$	$\exists x \forall y p(x, y)$	$\implies$	$\forall y \exists x p(x, y)$
T	T	T	T	T	T	T
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	F
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	F
F	F	F	F	F	T	F