２００３年度基礎数学講義ノート（２－１組）

２００３年9月２７日分

５章．連立 1 次方程式

$x_1, \dots, x_n$ を未知数とする連立 1 次方程式

$$\left\{\begin{array}{ccccccc} a_{11}x_1 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ & & \vdots & & & & \\ a_{n1}x_1 & + & \cdots & + & a_{nn}x_n & = & b_n \end{array}\right.$$

で $n = 2, 3$ の場合は中高で扱い，代表的な解法は加減法と代入法（まとめて消去法という）であった．消去法では未知数までかいていたが，未知数を省いて係数だけで解を求める方法を掃出法という．以下，簡単な場合について消去法と掃出法を比較してみる．

掃出法

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 2 & 3 & 1 & 7 \\ 3 & -1 & 2 & -1 \end{array}\right)$$

$$\downarrow \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & -5 & 3 \\ 0 & -7 & -7 & -7 \end{array}\right)$$

$$\downarrow \begin{array}{l} -R_2 \\ -\frac{1}{7}R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 5 & -3 \\ 0 & 1 & 1 & 1 \end{array}\right)$$

$$\downarrow \ R_3 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & -4 & 4 \end{array}\right)$$

$$\downarrow \ -\tfrac{1}{4}R_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\downarrow \begin{array}{l} R_1 - 3R_3 \\ R_2 - 5R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

消去法

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & + & 3z & = & 2 & \cdots ① \\ 2x & + & 3y & + & z & = & 7 & \cdots ② \\ 3x & - & y & + & 2z & = & -1 & \cdots ③ \end{array}\right.$$

$$\downarrow \begin{array}{l} ② - ① \times 2 \\ ③ - ① \times 3 \end{array}$$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & + & 3z & = & 2 & \cdots ① \\ & - & y & - & 5z & = & 3 & \cdots ②' \\ & - & 7y & - & 7z & = & -7 & \cdots ③' \end{array}\right.$$

$$\downarrow \begin{array}{l} ②' \times (-1) \\ ③' \times (-\frac{1}{7}) \end{array}$$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & + & 3z & = & 2 & \cdots ① \\ & & y & + & 5z & = & -3 & \cdots ②' \\ & & y & + & z & = & 1 & \cdots ③' \end{array}\right.$$

$$\downarrow \ ③' - ②'$$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & + & 3z & = & 2 & \cdots ① \\ & & y & + & 5z & = & -3 & \cdots ②' \\ & & & - & 4z & = & 4 & \cdots ③'' \end{array}\right.$$

$$\downarrow \ ③'' \times (-\tfrac{1}{4})$$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & + & 3z & = & 2 & \cdots ① \\ & & y & + & 5z & = & -3 & \cdots ②' \\ & & & & z & = & -1 & \cdots ③'' \end{array}\right.$$

$$\downarrow \begin{array}{l} ① - ③'' \times 3 \\ ②' - ③'' \times 5 \end{array}$$

$$\left\{\begin{array}{rcrcrcrl} x & + & 2y & & & = & 5 & \cdots ①' \\ & & y & & & = & 2 & \cdots ②'' \\ & & & & z & = & -1 & \cdots ③'' \end{array}\right.$$

$$\downarrow\ R_1-2R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\downarrow\ ①' - ②'' \times 2$$

$$\begin{cases} x & & & = & 1 \\ & y & & = & 2 \\ & & z & = & -1 \end{cases}$$

$\boxed{\text{掃出法}}$

$A\boldsymbol{x}=\boldsymbol{b}$ を

$$(A \mid \boldsymbol{b}) \xrightarrow{\text{行変形}} (E \mid \boldsymbol{x})$$

によって解く方法．ここで，行変形とは

(1) $kR_i \quad (k \neq 0)$

(2) $R_i \leftrightarrow R_j$

(3) $R_i + kR_j \quad$ ($k=0$ でもよい．$k=0$ のときは何もしていない！)

演習 5A

3. (1) $\begin{cases} x_1 + x_2 - x_3 = 0 \\ 2x_1 - 3x_2 + 5x_3 = -3 \\ 3x_1 + 6x_2 - 2x_3 = 7 \end{cases} \iff \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 5 \\ 3 & 6 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 7 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -3 & 5 & -3 \\ 3 & 6 & -2 & 7 \end{array}\right) \xrightarrow[R_3-3R_1]{R_2-2R_1} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 7 & -3 \\ 0 & 3 & 1 & 7 \end{array}\right) \xrightarrow{R_2+2R_3} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 9 & 11 \\ 0 & 3 & 1 & 7 \end{array}\right)$$

$$\xrightarrow[R_3-3R_2]{R_1-R_2} \left(\begin{array}{ccc|c} 1 & 0 & -10 & -11 \\ 0 & 1 & 9 & 11 \\ 0 & 0 & -26 & -26 \end{array}\right) \xrightarrow{-\frac{1}{26}R_3} \left(\begin{array}{ccc|c} 1 & 0 & -10 & -11 \\ 0 & 1 & 9 & 11 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[R_2-9R_3]{R_1+10R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \qquad \therefore \begin{cases} x_1 = -1 \\ x_2 = 2 \\ x_3 = 1 \end{cases}$$

$$※ \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -3 & 5 & -3 \\ 3 & 6 & -2 & 7 \end{array}\right) \xrightarrow[R_3-3R_1]{R_2-2R_1} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 7 & -3 \\ 0 & 3 & 1 & 7 \end{array}\right) \xrightarrow{5R_3+3R_2} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 7 & -3 \\ 0 & 0 & 26 & 26 \end{array}\right)$$

$$\xrightarrow{\frac{1}{26}R_3} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 7 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[R_2-7R_3]{R_1+R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -5 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-\frac{1}{5}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{R_1-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \qquad \therefore \begin{cases} x_1 = -1 \\ x_2 = 2 \\ x_3 = 1 \end{cases}$$