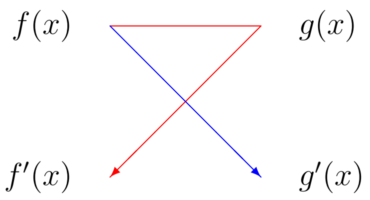
２００３年度基礎数学講義ノート（３－１組）

２００３年１２月１３日分

部分積分法（積の微分法則の逆）

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

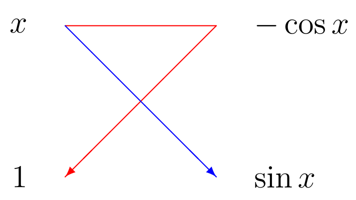
図式化



問題７．２

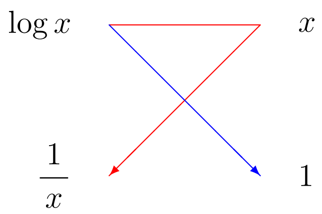
(1) 部分積分法より

$$\begin{aligned}\int x\sin x dx &= -x\cos x + \int \cos x dx \\ &= -x\cos x + \sin x\end{aligned}$$



(3) 部分積分法より

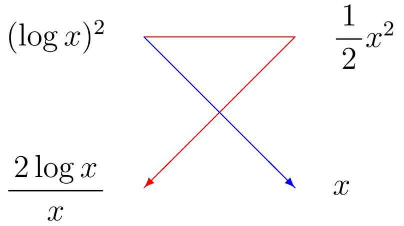
$$\begin{aligned}\int \log x dx &= x\log x - \int dx \\ &= x\log x - x\end{aligned}$$

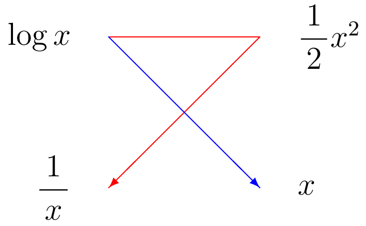


(4) 部分積分法より

$$\int x(\log x)^2 dx = \frac{1}{2}(x\log x)^2 - \int x\log x dx = \frac{1}{2}(x\log x)^2 - \left(\frac{1}{2}x^2\log x - \frac{1}{2}\int x dx\right)$$

$$= \frac{1}{2}(x\log x)^2 - \frac{1}{2}x^2\log x + \frac{1}{4}x^4$$





置換積分法（合成関数の微分法則の逆）

$$\int f(x)dx = \int f(g(t)) \cdot g'(t)dt \qquad (x = g(t))$$

特に

$$\int f(x)^\alpha \cdot f'(x)dx = \frac{1}{\alpha+1} f(x)^{\alpha+1} \quad (\alpha \neq -1), \qquad \int \frac{f'(x)}{f(x)}dx = \log|f(x)|$$

※ $x = g(t)$ より $\dfrac{dx}{dt} = g'(t)$ で，これを形式的に表すと

$$dx = g'(t)dt$$

これと $x = g(t)$ を左辺に「代入」することにより得られる．

※ $y = f(x)$ とおくと $dy = f'(x)dx$

$$\therefore \quad \int f(x)^\alpha \cdot f'(x)dx = \int y^\alpha dy = \frac{1}{\alpha+1} y^{\alpha+1} = \frac{1}{\alpha+1} f(x)^{\alpha+1}$$

$$\int \frac{f'(x)}{f(x)}dx = \int \frac{1}{f(x)} \cdot f'(x)dx = \int \frac{1}{y}dy = \log|y| = \log|f(x)|$$

問題７．１

(23) $\displaystyle \int \frac{3x^2}{x^3+1}dx = \int \frac{(x^3+1)'}{(x^3+1)}dx = \log|x^3+1|$

(24) $\displaystyle \int \frac{x+1}{x^2+2x+3} = \frac{1}{2}\int \frac{2x+2}{x^2+2x+3} = \frac{1}{2}\int \frac{(x^2+2x+3)'}{x^2+2x+3} = \log(x^2+2x+3)$

(25) $\displaystyle \int \frac{1}{\tan x}dx = \int \frac{\cos x}{\sin x}dx = \int \frac{(\sin x)'}{\sin x}dx = \log|\sin x|$

(27) $\displaystyle \int \frac{e^{2x}}{(e^x+1)^2}dx = \int \frac{e^x(e^x+1) - e^x}{(e^x+1)^2}dx = \int \left\{ \frac{e^x}{e^x+1} - \frac{e^x}{(e^x+1)^2} \right\} dx$

$\displaystyle = \int \left\{ \frac{(e^x+1)'}{e^x+1} - (e^x+1)^{-2} \cdot (e^x+1)' \right\} dx = \log(e^x+1) - \frac{1}{-2+1}(e^x+1)^{-2+1}$

$\displaystyle = \log(e^x+1) + \frac{1}{e^x+1}$