科目名	数学A	対象	1i-2	学部 研究科	工学部第二部	学科 専攻科		学籍 番号		評点
平成 26 年 1 月 21 日 (火) 3 回目 (～ 時限目)				担当	石川 学	学年		氏名		
試験 時間	60 分	注意 事項	① 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )							

2013 年度 I 科 2 組　後期試験

※解答用紙の裏面使用可

$\boxed{1}$ (1) 次の等式が成り立つような定数 A, B, C の値を求めよ．**(1) は答のみでよい．**

$$\frac{-x^2-27x-29}{(x-4)(x^2+6x+11)}=\frac{A}{x-4}+\frac{Bx+C}{x^2+6x+11}$$

(2) $\displaystyle\int \frac{-x^2-27x-29}{(x-4)(x^2+6x+11)}dx$ を求めよ．

$\boxed{2}$ $\sqrt{4x^2+5x+4}+2x=t$ とおくことにより，$\displaystyle\int \frac{2}{x\sqrt{4x^2+5x+4}}dx$ を求めよ．

$\boxed{3}$ $f(x,y)=x^3+xy^2+2x^2+y^2-2xy+x-2y$ について，次の問いに答えよ．

(1) $f(x,y)$ の停留点を求めよ．**(1) は答のみでよい．**

(2) $f(x,y)$ の極値を求めよ．

※ $f_x(a,b)=0,\ f_y(a,b)=0$ のとき
$H(a,b)>0,\ f_{xx}(a,b)>0 \implies f(a,b)$: 極小値
$H(a,b)>0,\ f_{xx}(a,b)<0 \implies f(a,b)$: 極大値
$H(a,b)<0 \implies f(a,b)$: 極値でない
ただし $H(x,y)=f_{xx}(x,y)f_{yy}(x,y)-f_{xy}(x,y)^2$ とする．

$\boxed{4}$ 次の積分を求めよ．

(1) $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} 2x\arctan x dx$　　(2) $\displaystyle\int_{-1}^{2}\left\{\int_{-1}^{x^2}(2x-7y)dy\right\}dx$

(3) $\displaystyle\int_{1}^{5}\left(\int_{1}^{x}\log y dy\right)dx$　　(4) $\displaystyle\int_{0}^{6}\left\{\int_{\frac{2}{3}x}^{4} y(y^3+36)^{-\frac{3}{2}}dy\right\}dx$　（順序変更）

(5) $\displaystyle\iint_D (7x-2y)dxdy$　　$(D : x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0)$

東京理科大学　平成　　年　　月　　日　**試　験　答　案**　　　　評　点

科目名		担当	先生
該当学部を○で囲む	理学部 1 / 工学部 2　部　学科　年　番	氏名	I－2

1

(1) 分母をはらうと

$$-x^2-27x-29 = A(x^2+6x+11)+(Bx+C)(x-4)$$

・$x=4 \cdots \ -153 = 51A \qquad \therefore A=-3$

・$x=0 \cdots \ -29 = 11A-4C$

$\quad -29 = -33-4C \qquad \therefore C=-1$

・$x=1 \cdots \ -57 = 18A-3(B+C)$

$\quad -19 = 6A-(B+C)$

$\quad -19 = -18-B+1 \qquad \therefore B=2$

よって $(A,B,C) = (-3,2,-1)$

(2) $\displaystyle\int \frac{-x^2-27x-29}{(x-4)(x^2+6x+11)}dx$

$$= \int\left(-\frac{3}{x-4}+\frac{2x-1}{x^2+6x+11}\right)dx$$

$$= \int\left\{-\frac{3}{x-4}+\frac{2x+6}{x^2+6x+11}-\frac{7}{(\sqrt{2})^2+(x+3)^2}\right\}dx$$

$$= -3\log|x-4|+\log(x^2+6x+11)-\frac{7}{\sqrt{2}}\arctan\frac{x+3}{\sqrt{2}}$$

2

$\sqrt{4x^2+5x+4} = t-2x$ の両辺を2乗すると

$$4x^2+5x+4 = t^2-4tx+4x^2$$

$$(4t+5)x = t^2-4 \qquad \therefore x = \frac{t^2-4}{4t+5}$$

また

$$\frac{dx}{dt} = \frac{2t\cdot(4t+5)-(t^2-4)\cdot 4}{(4t+5)^2} = \frac{2(2t^2+5t+8)}{(4t+5)^2}$$

さらに

$$\sqrt{4x^2+5x+4} = t-\frac{2(t^2-4)}{4t+5} = \frac{2t^2+5t+8}{4t+5}$$

よって

$$\int\frac{2}{x\sqrt{4x^2+5x+4}}dx = \int\frac{2}{\frac{t^2-4}{4t+5}\cdot\frac{2t^2+5t+8}{4t+5}}\cdot\frac{2(2t^2+5t+8)}{(4t+5)^2}dt$$

$$= \int\frac{4}{t^2-4}dt$$

$$= \int\frac{4}{(t-2)(t+2)}dt$$

$$= \int\left(\frac{1}{t-2}-\frac{1}{t+2}\right)dt$$

$$= \log|t-2|-\log|t+2|$$

3

$$f = x^3+xy^2+2x^2+y^2-2xy+x-2y$$

(1) $\begin{cases} f_x = 3x^2+y^2+4x-2y+1 = 0 & \text{—①} \\ f_y = 2xy+2y-2x-2 = 0 & \text{—②} \end{cases}$

②より $\quad 2(x+1)y-2(x+1) = 0$

$\quad 2(x+1)(y-1) = 0 \qquad \therefore x=-1 \text{ or } y=1$

$x=-1$ のとき，①より

$\quad 3+y^2-4-2y+1 = 0$

$\quad y(y-2) = 0 \qquad \therefore y=0,2$

$y=1$ のとき，①より

$\quad 3x^2+1+4x-2+1 = 0$

$\quad x(3x+4) = 0 \qquad \therefore x=0,-\frac{4}{3}$

よって，停留点は

$\quad (-1,0),\ (-1,2),\ (0,1),\ \left(-\frac{4}{3},1\right)$

(2) $f_{xx} = 6x+4$，$f_{yy} = 2x+2$，$f_{xy} = 2y-2$

$H = (6x+4)(2x+2)-(2y-2)^2$

・$H(-1,0) = (-2)\cdot 0-(-2)^2 = -4<0$

$\quad \therefore f(-1,0)$：極値でない

・$H(-1,2) = (-2)\cdot 0-2^2 = -4<0$

$\quad \therefore f(-1,2)$：極値でない

・$H(0,1) = 4\cdot 2-0^2 = 8>0$，$f_{xx}(0,1) = 4>0$

$\quad \therefore f(0,1) = -1$：極小値

・$H\left(-\frac{4}{3},1\right) = (-4)\cdot\left(-\frac{2}{3}\right)-0^2 = \frac{8}{3}>0$，$f_{xx}\left(-\frac{4}{3},1\right) = -4<0$

$\quad \therefore f\left(-\frac{4}{3},1\right) = \frac{5}{27}$：極大値

$\boxed{4}$

(1) $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} 2x\arctan x\,dx$

$\arctan x \quad x^2+1$

$\frac{1}{1+x^2} \quad 2x$

$\displaystyle = \Big[(x^2+1)\arctan x - x\Big]_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$\displaystyle = \left\{4\cdot\frac{\pi}{3} - \frac{4}{3}\cdot\left(-\frac{\pi}{6}\right)\right\} - \left\{\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)\right\}$

$\displaystyle = \frac{14}{9}\pi - \frac{4}{\sqrt{3}}$

(2) $\displaystyle\int_{-1}^{2}\left\{\int_{-1}^{x^2}(2x-7y)\,dy\right\}dx$

$\displaystyle = \int_{-1}^{2}\Big[2xy - \frac{7}{2}y^2\Big]_{y=-1}^{y=x^2}dx$

$\displaystyle = \int_{-1}^{2}\Big[2x\{x^2-(-1)\} - \frac{7}{2}(x^4-1)\Big]dx$

$\displaystyle = \int_{-1}^{2}\left(-\frac{7}{2}x^4 + 2x^3 + 2x + \frac{7}{2}\right)dx$

$\displaystyle = \Big[-\frac{7}{10}x^5 + \frac{1}{2}x^4 + x^2 + \frac{7}{2}x\Big]_{-1}^{2}$

$\displaystyle = -\frac{7}{10}\{32-(-1)\} + \frac{1}{2}(16-1) + (4-1) + \frac{7}{2}\{2-(-1)\}$

$\displaystyle = -\frac{21}{10}$

(3) $\displaystyle\int_{1}^{5}\left(\int_{1}^{x}\log y\,dy\right)dx$

$\displaystyle = \int_{1}^{5}\Big[y\log y - y\Big]_{y=1}^{y=x}dx$

$\displaystyle = \int_{1}^{5}\{(x\log x - 1\cdot 0) - (x-1)\}dx$

$\log x \quad \frac{1}{2}x^2$

$\frac{1}{x} \quad x$

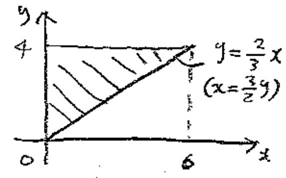
$\displaystyle = \int_{1}^{5}\{x\log x - (x-1)\}dx$

$\displaystyle = \Big[\frac{1}{2}x^2\log x - \frac{1}{4}x^2 - \frac{1}{2}(x-1)^2\Big]_{1}^{5}$

$\displaystyle = \frac{1}{2}(25\log 5 - 1\cdot 0) - \frac{1}{4}(25-1) - \frac{1}{2}(16-0)$

$\displaystyle = \frac{25}{2}\log 5 - 14$

(4) $\displaystyle\int_{0}^{6}\left\{\int_{\frac{2}{3}x}^{4} y(y^3+36)^{-\frac{3}{2}}dy\right\}dx$



$\displaystyle = \int_{0}^{4}\left\{\int_{0}^{\frac{3}{2}y} y(y^3+36)^{-\frac{3}{2}}dx\right\}dy$

$\displaystyle = \int_{0}^{4}\Big[xy(y^3+36)^{-\frac{3}{2}}\Big]_{x=0}^{x=\frac{3}{2}y}dy$

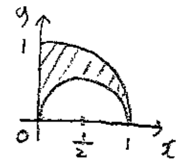
$\displaystyle = \int_{0}^{4}\frac{3}{2}y^2(y^3+36)^{-\frac{3}{2}}dy$

$\displaystyle = \frac{1}{2}\int_{0}^{4}(y^3+36)^{-\frac{3}{2}}\cdot 3y^2\,dy$

$\displaystyle = \frac{1}{2}\Big[-2(y^3+36)^{-\frac{1}{2}}\Big]_{0}^{4}$

$\displaystyle = -\left(\frac{1}{10} - \frac{1}{6}\right)$

$\displaystyle = \frac{1}{15}$



(5) $\displaystyle\iint_D (7x-2y)\,dx\,dy$ $\quad (D: x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0)$

$\displaystyle = \iint_{D'}(7r\cos\theta - 2r\sin\theta)\cdot r\,dr\,d\theta$ $\quad (D': 0 \leqq \theta \leqq \frac{\pi}{2},\ \cos\theta \leqq r \leqq 1)$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\left\{\int_{\cos\theta}^{1} r^2(7\cos\theta - 2\sin\theta)\,dr\right\}d\theta$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\Big[\frac{1}{3}r^3(7\cos\theta - 2\sin\theta)\Big]_{r=\cos\theta}^{r=1}d\theta$

$\displaystyle = \int_{0}^{\frac{\pi}{2}}\frac{1}{3}(1-\cos^3\theta)(7\cos\theta - 2\sin\theta)\,d\theta$

$\displaystyle = \frac{1}{3}\int_{0}^{\frac{\pi}{2}}\{7\cos\theta - 2\sin\theta - 7\cos^4\theta - 2\cos^3\theta\cdot(-\sin\theta)\}d\theta$

$\displaystyle = \frac{1}{3}\left(7\cdot 1 - 2\cdot 1 - 7\cdot\frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} - \Big[\frac{1}{2}\cos^4\theta\Big]_{0}^{\frac{\pi}{2}}\right)$

$\displaystyle = \frac{1}{3}\left\{5 - \frac{21}{16}\pi - \frac{1}{2}(0-1)\right\}$

$\displaystyle = \frac{1}{3}\left(\frac{11}{2} - \frac{21}{16}\pi\right)$

$\displaystyle = \frac{11}{6} - \frac{7}{16}\pi$