積分計算（１）

【問題 1】
次の原始関数（不定積分）を求めよ.

(1) $\displaystyle\int \left(x^4 - \frac{2}{x} + \frac{3}{x^2}\right)dx$

(2) $\displaystyle\int \left(4x^3 + \frac{5}{x^6}\right)dx$

(3) $\displaystyle\int \frac{x^3 + 2x - 1}{x}dx$

(4) $\displaystyle\int \left(x + \frac{1}{x}\right)^3 dx$

(5) $\displaystyle\int \left(3 - \frac{5}{x}\right)\left(4x^2 - 1 - \frac{2}{x}\right)dx$

(6) $\displaystyle\int \left(4x - 6 + \frac{5}{x}\right)\left(7x - \frac{1}{x} + \frac{3}{2x^2}\right)dx$

(7) $\displaystyle\int \frac{(x-1)^3}{x^2}dx$

(8) $\displaystyle\int \frac{(2x+1)^2}{x^3}dx$

(9) $\displaystyle\int \frac{(x-1)^2(3x-1)}{x^2}dx$

(10) $\displaystyle\int \frac{x^2+2}{\sqrt{x}}dx$

(11) $\displaystyle\int \left(\sqrt{x} + \frac{1}{x}\right)^2 dx$

(12) $\displaystyle\int \frac{(2x - 3\sqrt{x} + 1)^2}{\sqrt{x}}dx$

(13) $\displaystyle\int \frac{(x-1)^2}{\sqrt[3]{x}}dx$

(14) $\displaystyle\int \frac{(\sqrt[3]{x^2} + \sqrt[3]{x})^2}{\sqrt[3]{x}}dx$

(15) $\displaystyle\int \frac{1}{(3x+1)^2}dx$

(16) $\displaystyle\int \left(\frac{2}{3}x - 1\right)^5 dx$

(17) $\displaystyle\int \frac{1}{1-3x}dx$

(18) $\displaystyle\int \frac{1}{\sqrt{2x+1}}dx$

(19) $\displaystyle\int \sqrt[3]{5x-2}\,dx$

(20) $\displaystyle\int \sqrt[4]{2x+5}\,dx$

(21) $\displaystyle\int \frac{x^3 - 3x^2 - 9x}{x+2}dx$

解答

(1) $\displaystyle\int\left(x^4-\frac{2}{x}+\frac{3}{x^2}\right)dx=\frac{1}{5}x^5-2\log|x|-\frac{3}{x}+C$

※ $\displaystyle\int\frac{1}{x^2}dx=\int x^{-2}dx=\frac{1}{-2+1}x^{-2+1}+C=-\frac{1}{x}+C$　　（これは公式としたい）

(2) $\displaystyle\int\left(4x^3+\frac{5}{x^6}\right)dx=\int(4x^3+5x^{-6})dx=x^4-x^{-5}+C\ \left(=x^4-\frac{1}{x^5}+C\right)$

(3) $\displaystyle\int\frac{x^3+2x-1}{x}dx=\int\left(x^2+2-\frac{1}{x}\right)dx=\frac{1}{3}x^3+2x-\log|x|+C$

(4) $\displaystyle\int\left(x+\frac{1}{x}\right)^3dx=\int\left(x^3+3x+\frac{3}{x}+\frac{1}{x^3}\right)dx=\int\left(x^3+3x+\frac{3}{x}+x^{-3}\right)dx$

$\displaystyle\quad=\frac{1}{4}x^4+\frac{3}{2}x^2+3\log|x|-\frac{1}{2}x^{-2}+C=\frac{1}{4}x^4+\frac{3}{2}x^2+3\log|x|-\frac{1}{2x^2}+C$

(5) $\displaystyle\int\left(3-\frac{5}{x}\right)\left(4x^2-1-\frac{2}{x}\right)dx=\int\left(12x^2-20x-3-\frac{1}{x}+\frac{10}{x^2}\right)dx$

$\displaystyle\quad=4x^3-10x^2-3x-\log|x|-\frac{10}{x}+C$

(6) $\displaystyle\int\left(4x-6+\frac{5}{x}\right)\left(7x-\frac{1}{x}+\frac{3}{2x^2}\right)dx=\int\left(28x^2-42x+31+\frac{12}{x}-\frac{14}{x^2}+\frac{15}{2x^3}\right)dx$

$\displaystyle\quad=\frac{28}{3}x^3-21x^2+31x+12\log|x|+\frac{14}{x}-\frac{15}{4x^2}+C$

(7) $\displaystyle\int\frac{(x-1)^3}{x^2}dx=\int\frac{x^3-3x^2+3x-1}{x^2}dx=\int\left(x-3+\frac{3}{x}-\frac{1}{x^2}\right)dx$

$\displaystyle\quad=\frac{1}{2}x^2-3x+3\log|x|+\frac{1}{x}+C$

(8) $\displaystyle\int\frac{(2x+1)^2}{x^3}dx=\int\frac{4x^2+4x+1}{x^3}dx=\int\left(\frac{4}{x}+\frac{4}{x^2}+x^{-3}\right)dx$

$\displaystyle\quad=4\log|x|-\frac{4}{x}-\frac{1}{2}x^{-2}+C=4\log|x|-\frac{4}{x}-\frac{1}{2x^2}+C$

(9) $\displaystyle\int\frac{(x-1)^2(3x-1)}{x^2}dx=\int\frac{3x^3-7x^2+5x-1}{x^2}dx=\int\left(3x-7+\frac{5}{x}-\frac{1}{x^2}\right)dx$

$\displaystyle\quad=\frac{3}{2}x^2-7x+5\log|x|+\frac{1}{x}+C$

(10) $\displaystyle\int\frac{x^2+2}{\sqrt{x}}dx=\int(x^{\frac{3}{2}}+2x^{-\frac{1}{2}})dx=\frac{2}{5}x^{\frac{5}{2}}+2\cdot2x^{\frac{1}{2}}+C$

$\displaystyle\quad=\frac{2}{5}x^{\frac{5}{2}}+4x^{\frac{1}{2}}+C\ \left(=\frac{2}{5}\sqrt{x^5}+4\sqrt{x}+C\right)$

(11) $\displaystyle\int \left(\sqrt{x}+\frac{1}{x}\right)^2 dx = \int \left(x+\frac{2}{\sqrt{x}}+\frac{1}{x^2}\right) dx = \int \left(x+2x^{-\frac{1}{2}}+\frac{1}{x^2}\right) dx$

$\displaystyle\quad = \frac{x^2}{2}+2\cdot 2x^{\frac{1}{2}}-\frac{1}{x}+C = \frac{x^2}{2}+4\sqrt{x}-\frac{1}{x}+C$

(12) $\displaystyle\int \frac{(2x-3\sqrt{x}+1)^2}{\sqrt{x}}dx = \int \frac{4x^2-12x^{\frac{3}{2}}+13x-6x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}}dx$

$\displaystyle\quad = \int (4x^{\frac{3}{2}}-12x+13x^{\frac{1}{2}}-6+x^{-\frac{1}{2}})dx = 4\cdot\frac{2}{5}x^{\frac{5}{2}}-6x^2+13\cdot\frac{2}{3}x^{\frac{3}{2}}-6x+2x^{\frac{1}{2}}+C$

$\displaystyle\quad = \frac{8}{5}x^{\frac{5}{2}}-6x^2+\frac{26}{3}x^{\frac{3}{2}}-6x+2x^{\frac{1}{2}}+C$

(13) $\displaystyle\int \frac{(x-1)^2}{\sqrt[3]{x}}dx = \int \frac{x^2-2x+1}{x^{\frac{1}{3}}}dx = \int \left(x^{\frac{5}{3}}-2x^{\frac{2}{3}}+x^{-\frac{1}{3}}\right) dx$

$\displaystyle\quad = \frac{3}{8}x^{\frac{8}{3}}-2\cdot\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{2}x^{\frac{2}{3}}+C = \frac{3}{8}x^{\frac{8}{3}}-\frac{6}{5}x^{\frac{5}{3}}+\frac{3}{2}x^{\frac{2}{3}}+C$

(14) $\displaystyle\int \frac{(\sqrt[3]{x^2}+\sqrt[3]{x})^2}{\sqrt[3]{x}}dx = \int \frac{(x^{\frac{2}{3}}+x^{\frac{1}{3}})^2}{x^{\frac{1}{3}}}dx = \int \frac{x^{\frac{4}{3}}+2x+x^{\frac{2}{3}}}{x^{\frac{1}{3}}}dx = \int (x+2x^{\frac{2}{3}}+x^{\frac{1}{3}})dx$

$\displaystyle\quad = \frac{1}{2}x^2+2\cdot\frac{3}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}+C = \frac{1}{2}x^2+\frac{6}{5}x^{\frac{5}{3}}+\frac{3}{4}x^{\frac{4}{3}}+C$

(15) $\displaystyle\int \frac{1}{(3x+1)^2}dx = -\frac{1}{3(3x+1)}+C$

(16) $\displaystyle\int \left(\frac{2}{3}x-1\right)^5 dx = \frac{3}{2}\cdot\frac{1}{6}\left(\frac{2}{3}x-1\right)^6+C = \frac{1}{4}\left(\frac{2}{3}x-1\right)^6+C$

(17) $\displaystyle\int \frac{1}{1-3x}dx = -\frac{1}{3}\log|1-3x|+C$

(18) $\displaystyle\int \frac{1}{\sqrt{2x+1}}dx = \int (2x+1)^{-\frac{1}{2}}dx = \frac{1}{2}\cdot 2(2x+1)^{\frac{1}{2}}+C = \sqrt{2x+1}+C$

(19) $\displaystyle\int \sqrt[3]{5x-2}\,dx = \int (5x-2)^{\frac{1}{3}}dx = \frac{1}{5}\cdot\frac{3}{4}(5x-2)^{\frac{4}{3}}+C = \frac{3}{20}(5x-2)^{\frac{4}{3}}+C$

(20) $\displaystyle\int \sqrt[4]{2x+5}\,dx = \int (2x+5)^{\frac{1}{4}}dx = \frac{1}{2}\cdot\frac{4}{5}(2x+5)^{\frac{5}{4}}+C = \frac{2}{5}(2x+5)^{\frac{5}{4}}+C$

(21) $\displaystyle\int \frac{x^3-3x^2-9x}{x+2}dx = \int \frac{(x+2)(x^2-5x+1)-2}{x+2}dx = \int \left(x^2-5x+1-\frac{2}{x+2}\right) dx$

$\displaystyle\quad = \frac{1}{3}x^3-\frac{5}{2}x^2+x-2\log|x+2|+C$