| 科目名 | 微積分学 | 対象 | 1OB-AB | 学部研究科 | 理学部第一部 | 学科専攻科 | | 学籍番号 | | 評点 |
|---|---|---|---|---|---|---|---|---|---|---|
| 平成 25 年 8 月 5 日 (月) 2 回目 ( ～ 時限目) | | | | 担当 | 石川 学 | 学年 | | 氏名 | | |
| 試験時間 | 60 分 | 注意事項 | 1. 筆記用具以外持込不可<br>~~2. 下記のみ参照・持込可~~<br>( ) | | | | | | | |

平成 25 年度前期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ (1) を微分せよ．また，(2) ～ (6) を求めよ．

(1) $\arctan \dfrac{x}{\sqrt{x^2+1}+1}$  (2) $\displaystyle\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$

(3) $\displaystyle\int 2x \arcsin x\, dx$  (4) $\displaystyle\int_{-1}^{1+\sqrt{2}} \arctan x\, dx$

(5) $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} (3x^2+1)(\arctan x)^2 dx$  (6) $\displaystyle\int \frac{1}{(x^2+1)^2} dx$

$\boxed{2}$ (1) 次の関数の Maclaurin（マクローリン）展開を 4 次以下の項まで求めよ．必要ならば，下の Maclaurin 展開を用いてよい．ただし，**係数は既約分数**にすること．

(i) $\dfrac{\cos x}{\sqrt{1-x}}$  (ii) $\sqrt{1-x+2x^2}$

(2) 極限値 $\displaystyle\lim_{x\to 0} \frac{1}{x^4}\left(\frac{\cos x}{\sqrt{1-x}} - \sqrt{1+x}\right)$ を求めよ．答のみでよい．

**★代表的な関数の Maclaurin 展開**

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots \quad (x \in \mathbb{R})$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \cdots \quad (-1 < x \leqq 1)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots \quad (-1 \leqq x \leqq 1)$$

$\boxed{1}$ 40

(1) $\dfrac{1}{2(x^2+1)}$ 5  $\dfrac{1+\sqrt{\ }}{\{(\sqrt{\ }+1)^2+x^2\}\sqrt{\ }}$ △3  まとめ足りない △2

(2) $-\dfrac{e}{2}$ 5  $\dfrac{e}{2}$ △3

(3) $\dfrac{1}{2}\left\{(2x^2-1)\arcsin x + x\sqrt{1-x^2}\right\}$ 5

(4) $\dfrac{\pi}{8}$ 3 $+ \dfrac{3\sqrt{2}}{8}\pi$ 3 $- \dfrac{1}{2}\log(2+\sqrt{2})$ 4

(5) $\dfrac{37}{27\sqrt{3}}\pi^2$ 4 $- \dfrac{14}{9}\pi$ 4 $+ \dfrac{4}{\sqrt{3}}$ 2

(6) $\dfrac{x}{2(x^2+1)} + \dfrac{1}{2}\arctan x$ 5

$\dfrac{1}{2}\left(\theta + \dfrac{1}{2}\sin 2\theta\right)$ △3

$x^2 \arcsin x + \dfrac{1}{2}\left(\theta + \dfrac{1}{2}\sin 2\theta\right) - \arcsin x$ △3

$(1+\sqrt{2})\arctan(1+\sqrt{2})$ △1 $- \dfrac{\pi}{4}$ △1 $- \dfrac{1}{2}\left[\log\{1+(1+\sqrt{2})^2\} - \log 2\right]$ △1

$\boxed{2}$ 20

(1) (i) $1$ 1 $+ \dfrac{1}{2}x$ 1 $- \dfrac{1}{8}x^2$ 1 $+ \dfrac{1}{16}x^3$ 1 $+ \dfrac{49}{384}x^4$ 3 $+ \cdots$  7

(ii) $1$ 1 $- \dfrac{1}{2}x$ 1 $+ \dfrac{7}{8}x^2$ 2 $+ \dfrac{7}{16}x^3$ 3 $- \dfrac{21}{128}x^4$ 3 $+ \cdots$  10

(2) $\dfrac{1}{6}$ 3  3

東京理科大学　年　月　日　**試験答案**　評点

| 科目名 | | 担当 | 先生 |
|---|---|---|---|
| 所属 学籍番号 | 学部　学科　番 | 氏名 | 40 + 20 |

評点: 60

$\boxed{1}$

(1) $\left(\arctan\frac{x}{\sqrt{x^2+1}+1}\right)' = \frac{1}{1+\left(\frac{x}{\sqrt{x^2+1}+1}\right)^2}\cdot\frac{1\cdot(\sqrt{x^2+1}+1)-x\cdot\frac{x}{\sqrt{x^2+1}}}{(\sqrt{x^2+1}+1)^2} = \frac{(\sqrt{\ }+1)^2}{x^2+1+2\sqrt{\ }+1+x^2}\cdot\frac{x^2+1+\sqrt{\ }-x^2}{(\sqrt{\ }+1)^2\sqrt{x^2+1}}$

$= \frac{\sqrt{\ }+1}{2\sqrt{\ }(\sqrt{\ }+1)\cdot\sqrt{\ }} = \frac{1}{2(x^2+1)}$

(2) $\lim_{x\to0}\frac{(1+x)^{\frac{1}{x}}-e}{x} = \lim_{x\to0}\frac{e^{\frac{1}{x}\log(1+x)}-e}{x} = \lim_{x\to0} e^{\frac{1}{x}\log(1+x)}\cdot\frac{\frac{1}{1+x}\cdot x-\log(1+x)\cdot 1}{x^2} = e^1\cdot\lim_{x\to0}\frac{x-(1+x)\log(1+x)}{x^2(1+x)}$

$= e\cdot\lim_{x\to0}\frac{x-(1+x)\log(1+x)}{x^2} = e\cdot\lim_{x\to0}\frac{1-\{1\cdot\log(1+x)+(1+x)\cdot\frac{1}{1+x}\}}{2x} = e\cdot\lim_{x\to0}\frac{-\log(1+x)}{2x} = e\cdot\left(-\frac{1}{2}\right) = -\frac{e}{2}$

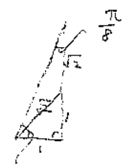
(3) $\int 2x\arcsin x\,dx = (x^2-1)\arcsin x+\int\sqrt{1-x^2}\,dx = (x^2-1)\arcsin x+\frac{1}{2}\left(x\sqrt{1-x^2}+\arcsin x\right)$

$= \frac{1}{2}\left\{(2x^2-1)\arcsin x+x\sqrt{1-x^2}\right\}$

$\arcsin x \quad x^2-1$
$\frac{1}{\sqrt{1-x^2}} \quad 2x$

(4) $\int_{-1}^{1+\sqrt{2}}\arctan x\,dx = \left[x\arctan x-\frac{1}{2}\log(1+x^2)\right]_{-1}^{1+\sqrt{2}} = \left\{(1+\sqrt{2})\cdot\frac{3}{8}\pi-(-1)\cdot\left(-\frac{\pi}{4}\right)\right\}-\frac{1}{2}\left\{\log(4+2\sqrt{2})-\log 2\right\}$

$= \frac{\pi}{8}+\frac{3\sqrt{2}}{8}\pi-\frac{1}{2}\log(2+\sqrt{2})$



$\frac{\pi}{2}-\frac{\pi}{8}=\frac{3}{8}\pi$
$1+2\sqrt{2}+2$

(5) $\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}}(3x^2+1)(\arctan x)^2\,dx = \left[x(x^2+1)(\arctan x)^2\right]_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} - \int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}}2x\arctan x\,dx$

$= \left\{\sqrt{3}\cdot4\cdot\frac{\pi^2}{9}-\left(-\frac{1}{\sqrt{3}}\right)\cdot\frac{4}{3}\cdot\frac{\pi^2}{36}\right\} - \left[(x^2+1)\arctan x - x\right]_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$= \frac{37}{27\sqrt{3}}\pi^2-\left\{4\cdot\frac{\pi}{3}-\frac{4}{3}\cdot\left(-\frac{\pi}{6}\right)\right\}+\left\{\sqrt{3}-\left(-\frac{1}{\sqrt{3}}\right)\right\}$

$= \frac{37}{27\sqrt{3}}\pi^2-\frac{14}{9}\pi+\frac{4}{\sqrt{3}}$

$(\arctan x)^2 \quad x^3+x=x(x^2+1)$
$\frac{2\arctan x}{1+x^2} \quad 3x^2+1$

$\arctan x \quad x^2+1$
$\frac{1}{1+x^2} \quad 2x$

(6) $\int\frac{1}{(x^2+1)^2}\,dx = \int\frac{(x^2+1)-x^2}{(x^2+1)^2}\,dx = \int\left\{\frac{1}{x^2+1}-\frac{x}{2}\cdot\frac{2x}{(x^2+1)^2}\right\}dx = \int\frac{1}{x^2+1}\,dx-\left\{-\frac{x}{2(x^2+1)}+\frac{1}{2}\int\frac{1}{x^2+1}\,dx\right\}$

$= \frac{x}{2(x^2+1)}+\frac{1}{2}\int\frac{1}{x^2+1}\,dx = \frac{x}{2(x^2+1)}+\frac{1}{2}\arctan x$

$\frac{x}{2} \quad -\frac{1}{x^2+1}$
$\frac{1}{2} \quad \frac{2x}{(x^2+1)^2}$

2

(1) (i)

$$\frac{\cos x}{\sqrt{1-x}} = \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots\right) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \cdots\right)$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \cdots$$

$$- \frac{1}{2}x^2 - \frac{1}{4}x^3 - \frac{3}{16}x^4 + \cdots$$

$$+ \frac{1}{24}x^4 + \cdots$$

$$+ \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \frac{49}{384}x^4 + \cdots$$

(ii)

$$\sqrt{1-x+2x^2} = 1 + \frac{1}{2}(-x+2x^2) - \frac{1}{8}(-x+2x^2)^2 + \frac{1}{16}(-x+2x^2)^3 - \frac{5}{128}(-x+2x^2)^4 + \cdots$$

$$= 1 + \frac{1}{2}(-x+2x^2) - \frac{1}{8}(x^2-4x^3+4x^4) + \frac{1}{16}(-x^3+6x^4+\cdots) - \frac{5}{128}(x^4 - \cdots) + \cdots$$

$$= 1 - \frac{1}{2}x + \frac{7}{8}x^2 + \frac{7}{16}x^3 - \frac{21}{128}x^4 + \cdots$$

(2)

$$\lim_{x\to 0}\frac{1}{x^4}\left(\frac{\cos x}{\sqrt{1-x}} - \sqrt{1+x}\right) = \lim_{x\to 0}\frac{1}{x^4}\left\{\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3+\frac{49}{384}x^4+o(x^3)\right) - \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+o(x^4)\right)\right\}$$

$$= \lim_{x\to 0}\frac{1}{x^4}\left(\frac{1}{6}x^4 + o(x^4)\right) = \lim_{x\to 0}\left(\frac{1}{6}+o(1)\right) = \frac{1}{6}$$