科目名	微積分学 B 微積分学	対 象	1OB-AB	学 部 研究科	理学部第一部	学 科 専攻科		学 籍 番 号	
平成 30 年 1 月 29 日 (月) 2 回目 (～ 時限目)				担 当	石川 学	学 年		氏 名	
試 験 時 間	60 分	注 意 事 項	1. 筆記用具以外持込不可 ~~2. 下記のみ参照・持込可~~ ( )						

評 点

平成 29 年度後期定期試験

※解答用紙の裏面使用可

$\boxed{1}$ 次の積分を求めよ. （80 点）

(1) $\displaystyle\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(7x-2y)dy\right\}dx$

(2) $\displaystyle\int_{e}^{e^2}\left\{\int_{1}^{x}\frac{(\log x)^3}{xy}dy\right\}dx$

(3) $\displaystyle\int_{-1}^{2}\left(\int_{-x}^{x+2}ye^{x}dy\right)dx$

(4) $\displaystyle\int_{1}^{5}\left(\int_{1}^{x}\log\frac{\sqrt{x}}{y}dy\right)dx$

(5) $\displaystyle\int_{0}^{4}\left(\int_{\frac{\sqrt{x}}{2}}^{1}xe^{-y^5}dy\right)dx$ （順序変更）

(6) $\displaystyle\int\int_{D}\frac{y^3}{x^6}dxdy \quad \left(D: 1 \leqq x^2+y^2 \leqq 4,\ -x \leqq y \leqq \sqrt{3}\,x\right)$

(7) $\displaystyle\int\int_{D}y(13x-8y)dxdy \quad \left(D: x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0\right)$

計算に必要なもの（抜粋）

(1) 部分積分

(2) 置換積分

$$\int \frac{f'(x)}{f(x)}dx = \log|f(x)|$$

$$\int f(x)^{\alpha} \cdot f'(x)dx = \frac{1}{\alpha+1}f(x)^{\alpha+1} \quad (\alpha \neq -1)$$

$$\int e^{f(x)} \cdot f'(x)dx = e^{f(x)}$$

(3) 公式

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \dfrac{\pi}{2} & (n=0) \\ 1 & (n=1) \\ \dfrac{(n-1)\cdot(n-3)\cdot \cdots \cdot 1}{n \cdot (n-2) \cdot \cdots \cdot 2} \cdot \dfrac{\pi}{2} & (n=2,4,6,\dots) \\ \dfrac{(n-1)\cdot(n-3)\cdot \cdots \cdot 2}{n \cdot (n-2) \cdot \cdots \cdot 3} \cdot 1 & (n=3,5,7,\dots) \end{cases}$$

$\boxed{1}$ 次の積分を求めよ. （80 点）

(1) $\displaystyle\int_{-1}^{2} \left\{ \int_{2x-1}^{x^2} (7x-2y)dy \right\} dx$

(2) $\displaystyle\int_{e}^{e^2} \left\{ \int_{1}^{x} \frac{(\log x)^3}{xy} dy \right\} dx$

(3) $\displaystyle\int_{-1}^{2} \left(\int_{-x}^{x+2} ye^x dy \right) dx$

(4) $\displaystyle\int_{1}^{5} \left(\int_{1}^{x} \log \frac{\sqrt{x}}{y} dy \right) dx$

(5) $\displaystyle\int_{0}^{4} \left(\int_{\frac{\sqrt{x}}{2}}^{1} xe^{-y^5} dy \right) dx$ （順序変更）

(6) $\displaystyle\int\int_D \frac{y^3}{x^6} dxdy \quad \left(D : 1 \leqq x^2+y^2 \leqq 4,\ -x \leqq y \leqq \sqrt{3}\,x \right)$

(7) $\displaystyle\int\int_D y(13x-8y)dxdy \quad \left(D : x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0 \right)$

★減点ポイント

(1) 答えの符号ミス → 各 2 点減点

(2) 約分忘れ → 各 1 点減点

(3) まとめ不足やミス → その程度により 1 点〜 5 点減点

解答

(1)
$$\begin{aligned}
\int_{-1}^{2}\left\{\int_{2x-1}^{x^2}(7x-2y)dy\right\}dx &= \int_{-1}^{2}\Big[7xy-y^2\Big]_{y=2x-1}^{y=x^2}dx \\
&= \int_{-1}^{2}\big[(7x^3-x^4)-\{7x(2x-1)-(2x-1)^2\}\big]dx \\
&= \int_{-1}^{2}(-x^4+7x^3-10x^2+3x+1)dx \\
&= \left[-\frac{1}{5}x^5+\frac{7}{4}x^4-\frac{10}{3}x^3+\frac{3}{2}x^2+x\right]_{-1}^{2} \\
&= \left(-\frac{32}{5}+28-\frac{80}{3}+6+2\right)-\left(\frac{1}{5}+\frac{7}{4}+\frac{10}{3}+\frac{3}{2}-1\right) \\
&= -\frac{57}{20} \qquad \text{（10 点）}
\end{aligned}$$

(2)
$$\begin{aligned}
\int_{e}^{e^2}\left\{\int_{1}^{x}\frac{(\log x)^3}{xy}dy\right\}dx &= \int_{e}^{e^2}\left[\frac{(\log x)^3}{x}\log y\right]_{y=1}^{y=x}dx \\
&= \int_{e}^{e^2}\frac{(\log x)^3}{x}(\log x-0)dx \\
&= \int_{e}^{e^2}(\log x)^4\cdot\frac{1}{x}dx \\
&= \left[\frac{1}{5}(\log x)^5\right]_{e}^{e^2} \\
&= \frac{1}{5}(32-1) \\
&= \frac{31}{5} \qquad \text{（10 点）}
\end{aligned}$$

(3)
$$\begin{aligned}
\int_{-1}^{2}\left(\int_{-x}^{x+2}ye^x dy\right)dx &= \int_{-1}^{2}\left[\frac{1}{2}y^2e^x\right]_{y=-x}^{y=x+2}dx \\
&= \int_{-1}^{2}\frac{1}{2}\{(x+2)^2-x^2\}e^x dx \\
&= \int_{-1}^{2}(2x+2)e^x dx \\
&= \Big[(2x+2)e^x-2e^x\Big]_{-1}^{2} \\
&= \Big[2xe^x\Big]_{-1}^{2} \\
&= 4e^2-(-2e^{-1}) \\
&= 4e^2+\frac{2}{e} \qquad \text{（順に 5 点, 5 点）}
\end{aligned}$$

(4) $$\begin{aligned}\int_1^5\left(\int_1^x \log\frac{\sqrt{x}}{y}dy\right)dx &= \int_1^5\left\{\int_1^x\left(\frac{1}{2}\log x-\log y\right)dy\right\}dx\\
&= \int_1^5\left[\frac{1}{2}y\log x-(y\log y-y)\right]_{y=1}^{y=x}dx\\
&= \int_1^5\left\{\frac{1}{2}(x-1)\log x-(x\log x-1\cdot 0)+(x-1)\right\}dx\\
&= \int_1^5\left\{-\frac{1}{2}x\log x-\frac{1}{2}\log x+(x-1)\right\}dx\\
&= \left[-\frac{1}{2}\left(\frac{1}{2}x^2\log x-\frac{1}{4}x^2\right)-\frac{1}{2}(x\log x-x)+\frac{1}{2}(x-1)^2\right]_1^5\\
&= -\frac{1}{4}(25\log 5-1\cdot 0)+\frac{1}{8}(25-1)-\frac{1}{2}(5\log 5-1\cdot 0)+\frac{1}{2}(5-1)+\frac{1}{2}(16-0)\\
&= 13-\frac{35}{4}\log 5 \qquad \text{（順に 5 点, 5 点）}\end{aligned}$$

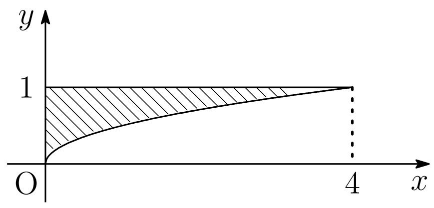
(5) 積分領域は

$D: 0 \leqq x \leqq 4,\ \dfrac{\sqrt{x}}{2} \leqq y \leqq 1$

であるが，これは

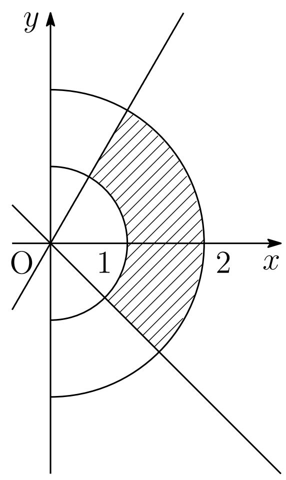
$D: 0 \leqq y \leqq 1,\ 0 \leqq x \leqq 4y^2$

でもあるから



$$\begin{aligned}\int_0^4\left(\int_{\frac{\sqrt{x}}{2}}^1 xe^{-y^5}dy\right)dx &= \int_0^1\left(\int_0^{4y^2} xe^{-y^5}dx\right)dy \qquad \left(\begin{array}{l}\text{順序変更が正しくできて 3 点}\\ \text{順序変更が正しくない場合，これ以降は見ない}\end{array}\right)\\
&= \int_0^1\left[\frac{1}{2}x^2e^{-y^5}\right]_{x=0}^{x=4y^2}dy\\
&= \int_0^1 8y^4e^{-y^5}dy\\
&= -\frac{8}{5}\int_0^1 e^{-y^5}\cdot(-5y^4)dy\\
&= -\frac{8}{5}\left[e^{-y^5}\right]_0^1\\
&= -\frac{8}{5}\left(\frac{1}{e}-1\right)\\
&= \frac{8}{5}\left(1-\frac{1}{e}\right) \qquad \text{（7 点）}\end{aligned}$$

(6) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{pmatrix} r \geqq 0 \\ \theta : 1\ \text{周分} \end{pmatrix}$ とおくと



$$\int\int_D \frac{y^3}{x^6}dxdy \quad \left(D : 1 \leqq x^2+y^2 \leqq 4,\ -x \leqq y \leqq \sqrt{3}\,x\right)$$

$$= \int\int_{D'} \frac{r^3\sin^3\theta}{r^6\cos^6\theta}\cdot rdrd\theta \quad \left(D' : 1 \leqq r \leqq 2,\ -\frac{\pi}{4} \leqq \theta \leqq \frac{\pi}{3}\right)$$

$$= \left(\int_1^2 \frac{1}{r^2}dr\right) \times \left(\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin^3\theta}{\cos^6\theta}d\theta\right)$$

$$= \left[-\frac{1}{r}\right]_1^2 \times \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(1-\cos^2\theta)\sin\theta}{\cos^6\theta}d\theta$$

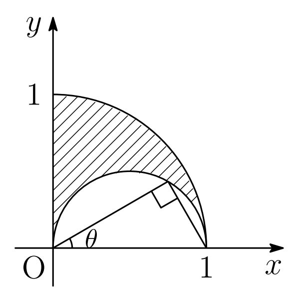
$$= -\left(\frac{1}{2}-1\right) \times \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \left\{-(\cos\theta)^{-6}\cdot(-\sin\theta) + (\cos\theta)^{-4}\cdot(-\sin\theta)\right\}d\theta$$

$$= \frac{1}{2} \times \left[\frac{1}{5}(\cos\theta)^{-5} - \frac{1}{3}(\cos\theta)^{-3}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left\{\frac{1}{5}(32-4\sqrt{2}) - \frac{1}{3}(8-2\sqrt{2})\right\}$$

$$= \frac{28-\sqrt{2}}{15}$$ （順に 7 点，8 点）

(7) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \begin{pmatrix} r \geqq 0 \\ \theta : 1\ \text{周分} \end{pmatrix}$ とおくと



$$\int\int_D y(13x-8y)dxdy \quad \left(D : x \leqq x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0\right)$$

$$= \int\int_{D'} r\sin\theta(13r\cos\theta - 8r\sin\theta)\cdot rdrd\theta \quad \left(D' : 0 \leqq \theta \leqq \frac{\pi}{2},\ \cos\theta \leqq r \leqq 1\right)$$

$$= \int\int_{D'} r^3\sin\theta(13\cos\theta - 8\sin\theta)drd\theta$$

$$= \int_0^{\frac{\pi}{2}} \left\{\int_{\cos\theta}^1 r^3\sin\theta(13\cos\theta-8\sin\theta)dr\right\}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{4}r^4\sin\theta(13\cos\theta-8\sin\theta)\right]_{r=\cos\theta}^{r=1}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{4}(1-\cos^4\theta)\sin\theta(13\cos\theta-8\sin\theta)d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \{13(\cos\theta\sin\theta - \cos^5\theta\sin\theta) - 8(\sin^2\theta - \cos^4\theta\sin^2\theta)\}d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left[13\{-\cos\theta\cdot(-\sin\theta) + \cos^5\theta\cdot(-\sin\theta)\} - 8(\sin^2\theta - \cos^4\theta + \cos^6\theta)\right]d\theta$$

$$= \frac{1}{4}\left\{13\left[-\frac{1}{2}\cos^2\theta + \frac{1}{6}\cos^6\theta\right]_0^{\frac{\pi}{2}} - 8\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} + \frac{5\cdot 3\cdot 1}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}\right)\right\}$$

$$= \frac{1}{4}\left[13\left\{-\frac{1}{2}(0-1) + \frac{1}{6}(0-1)\right\} - \frac{7}{4}\pi\right]$$

$$= \frac{13}{12} - \frac{7}{16}\pi$$ （順に 7 点，8 点）